

A Steep Learning Curve

<http://cba.unomaha.edu/faculty/mohara/web/AEF-2012-Steep-Learning-Curve.pdf>

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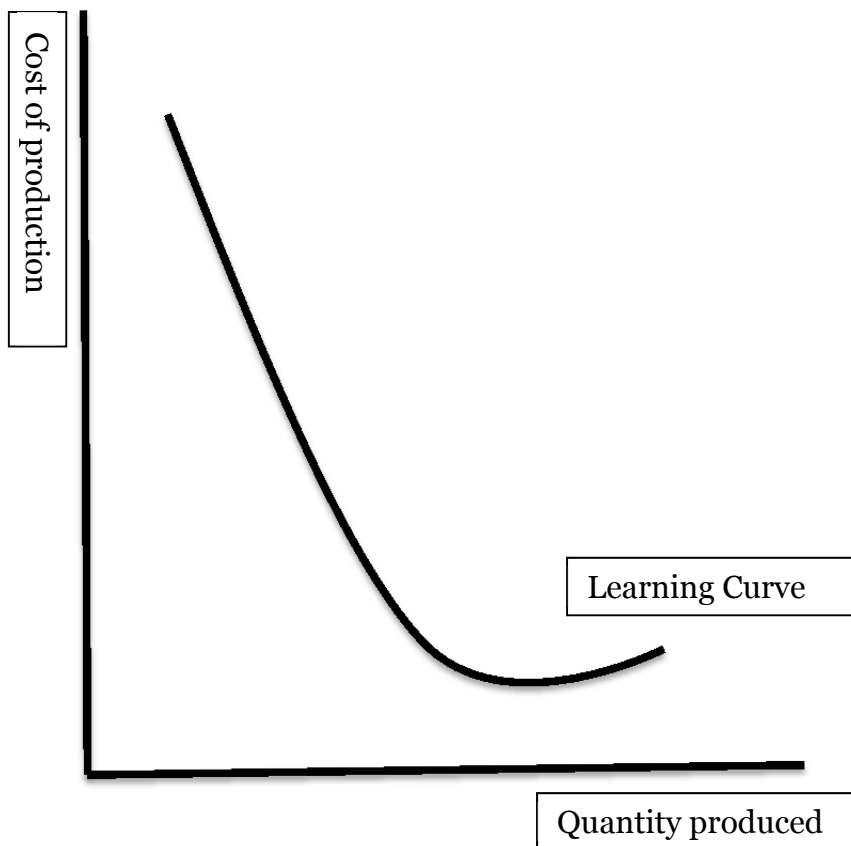
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ABSTRACT: *Even inattentive business students have heard, and quite likely have used, the phrase "a steep learning curve." However, rarely is that phrase used correctly. In the vernacular "a steep learning curve" often appears to mean little more than a lot of learning in short period of time; and, it might or might not refer to the work needed to learn. Technically, the relationship of The Learning Curve is far more precise (i.e., $\% \Delta \text{ Ave Cost per } 2^n Q$). This pedagogy manuscript will discuss four items. [1] It will contrast how not to as well as how to draw The Learning Curve. [2] It will explore some of the mathematical attributes of The Learning Curve graph and uses of the curve. [3] It will identify a few glaring errors routinely made by even knowledgeable persons when drawing a learning curve. [4] It will propose an alternative graphic presentation, the knowledge curve, that actually focuses upon learning versus cost of production.*

Graph #1 is on the next page and is a striped down Learning Curve. Note especially that it really is a cost of production graph. Note also that any representation of learning is an asserted implication; and that learning is but one of many plausible asserted implications for the shape of this curve. Is "learning" the only production input (!) that labors under the law of diminishing marginal returns or benefits from economies of size?¹ No wonder our students get confused. Economics does this a lot. Economics often names a graph for an implication of the interaction of the independent variable with the dependent variable rather than for the variables involved. For example, the demand curve really is a price and quantity demanded curve. And, in turn countless students struggle with the critical distinction between a change in demand (i.e., ΔD = movement **of** the curve) versus a change in quantity demanded (i.e., ΔQ_d = movement **on** the curve).

¹ Recall that economies of scale require proportional increases in *all* inputs whereas economies of size only require proportional increases in *most* inputs. If learning is to be a production input (rather than an output), then the steepness of the Learning Curve means that learning is not increasing proportionally.

**Graph #1: A STRIPED DOWN LEARNING CURVE;
IT AVOIDS CLEAR ERROR,
BUT INVITES STUDENT CONFUSION**



Let's recall Adam Smith's identification of three sources of gains from specializing labor inputs via the division of labor. To wit: [i] differences in natural **ability**; [ii] avoidance of **time** expenditures as shift from one task to another task; and, of interest to us here, [iii] **learning**. Repetitive production creates the opportunity to learn more efficient production techniques, both by the workers form of labor and by the managerial form of labor.

Such learning can range across the continuum from mere muscle memory to radical reformulations of the production process. Each form of learning input, within particular ranges of production, might generate either a gentle or might generate a steep slope for the Learning Curve. However, the mixture of learning inputs necessarily must shift towards a much greater contribution from reformulations of the production processes as cumulative output continues to double.²

Let's look at the Learning Curve from the student's perspective, and let's do so using two sentences: [A] and [B], both of which are true. Sentence [A] reads: *The Learning Curve seeks to portray changes in the average cost of production with increases in the cumulative output quantity of production.* From your student's perspective, think about how complex that sentence is. Note particularly how many variables are in play as well as how many arithmetic equations are implied by that sentence. Now, let's push on that sentence a little harder. Sentence [B] reads: *Technically, the focus of the Learning Curve is percentage changes in the average cost of production as the cumulative output is doubled.* We just shifted our unit of analysis from \$1 and 1 widget to units of analysis that are $\$1.0 \times 10^n$ of cost and 1.0×2^n units of widgets. Is it any surprise that our students do not notice the magnitude of shift from sentence [A] to sentence [B]? The Learning Curve is simple, yes, it is beguilingly simple.

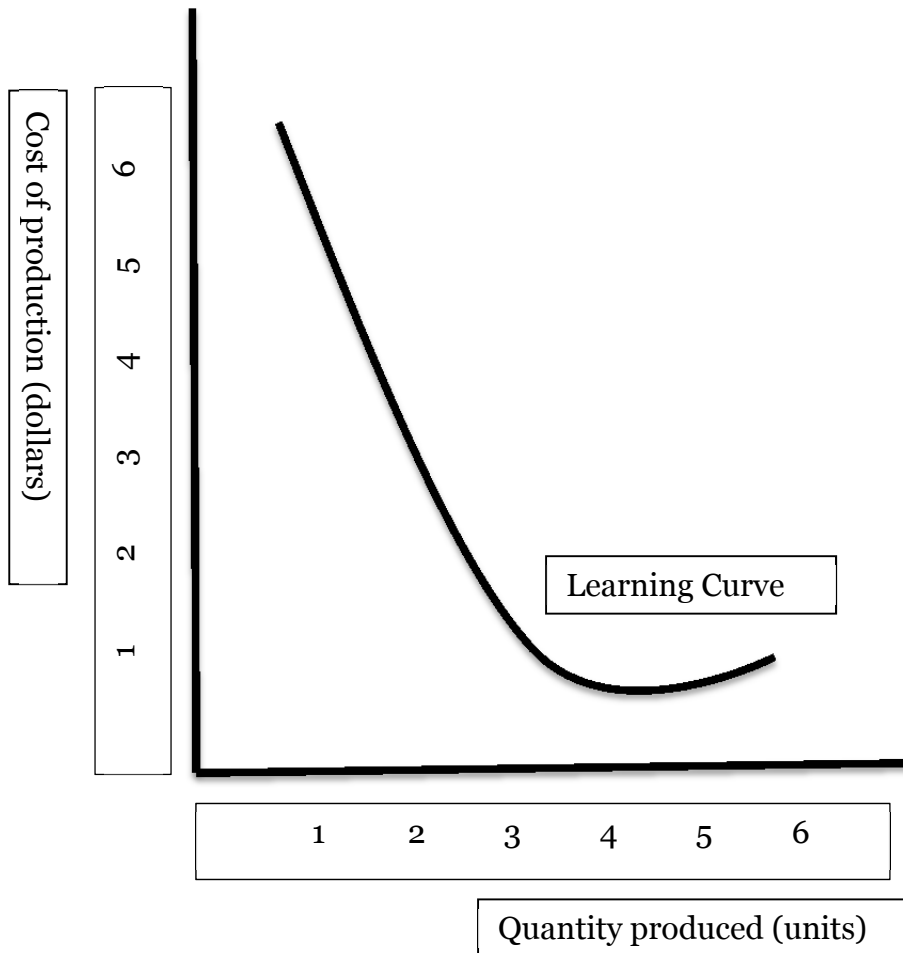
Graph #1, above with title "A Striped Down Learning Curve; it Avoids Clear Error, but Invites Student Confusion" is accurate because of what it omits.

Graph #1's omissions especially invite student error because of those omissions are far from ordinary. When a graph is drawn quickly it is customary to leave out ordinary elements of the graph. Thus, Graph #1's omissions invite a student to supply the ordinary elements when filling in Graph #1's omissions. That is the invited error since what Graph #1 omits is not ordinary. Graph #1's horizontal axis is true as far as it goes. So too is Graph #1's vertical axis: true as far as it goes. The problem is what is missing is not as ordinarily would be expected from a graph. Saying nothing avoids saying something wrong; but, also quickly leads students astray. Be honest. When you look at Graph #1 do you remember the Learning Curve is $\% \Delta \text{ Ave Cost per } 2^n \text{ Q}$. OK, you are a Ph.D., but did you also recall to insert the correct units on the horizontal axis and the correct units on the vertical axis?

Below, let's look at Graph #2: A Learning Curve as Routinely Done: that is, As Done Wrong". One of two things necessarily is wrong with Graph #2. Either the units on each axis is right and the curve is wrong; or, the units are wrong and the curve is right. Well, since the phrase is a "steep learning curve" it is likely that the error lies in the units. Graph #1 is flawed because neither axis has numeric units. Graph #1 invites the error of filling in its omission incorrectly. The routine error that is made when filling in Graph #1's omission is what is portrayed in Graph #2.

² Scalar changes associated with a doubling of cumulative output invites the conclusion of a need for a fundamental reformulation of the production process. Cumulative total output can be doubled using constant technology. However, to do so requires many more production cycles (read: time). More time, in turn, requires stable demand for the output. Doubling the time with constant technology with double the cumulative output, but it is unlikely that market demand will be that stable (but possible, for example, demand for high quality handmade Swiss watches). If demand stability does not exist, then the firm would cease to exist prior to achieving the next doubling of cumulative total output. In this day and age, such demand stability and such stable production technology would be odd.

**Graph #2: A LEARNING CURVE AS ROUTINELY DONE:
that is, AS DONE WRONG**



Also, let's think a moment about the curve drawn on Graph #2 with the numerical units of Graph #2. At one unit of output the cost is about \$6. At two units of output the cost is about \$3. At four units of output the cost is minimized at about \$0.50. Talk about steep. A 50% savings on the first doubling and an over 80% savings on the second doubling. Well, it's hard to look a gift horse in the mouth, so *that* steepness ought to be excused as an insufficient warning to a student already led astray by the omission from Graph #1.

Below is Graph #3: Learning Curve Drawn Correctly. On Graph #3 the Learning Curve's horizontal axis is a specific measure of quantity produced. The ordinary measure of quantity has a horizontal axis that counts off increments of units of quantity. That was shown in Graph #2, above. For example, an ordinary quantity axis would read 1 unit, 2 units, 3 units, etc. Not so the horizontal axis of the Learning Curve. For the Learning Curve the metric for quantity is a doubling of output. That is, the horizontal axis of the Learning Curve does count off 1, 2, 3, 4; but, those are doublings. Or, more mathematically, powers of 2. As shown below in Graph #3, the Learning Curve

horizontal axis counts off $2^0, 2^1, 2^2, 2^3, 2^4$, etc. Recall that any number raised to the zero power is 1 and any number raised to the power of 1 is itself. For example,

2^0 equals one and 2^1 equals two. That is, on the Learning Curve the horizontal axis the equal horizontal distances count off doublings of cumulative output units. To wit: equal horizontal distances count off as 1, 2, 4, 8, 16, etc. of cumulative total output.³

The vertical axis also is radically different from what is expected, but in a materially different way than is the horizontal axis.

The Learning Curve seeks to explain changes in average cost of production. Specifically, the Learning Curve seeks to explain **percentage changes** in the average cost of production. Accordingly, the vertical axis of the learning curve also counts off in powers, but this time in powers of 10. That is, the Learning Curve vertical axis does not count off \$1, \$2, \$3. Instead, its equal distant vertical markers counts off

$10^0, 10^1, 10^2, 10^3, 10^4$; or, if you prefer, \$1, \$10, \$100, \$1,000, and \$10,000. Using base 10 raised to a power means that *equal vertical distances* on the axis measure out *equal percentage changes*.

In the striped down Learning Curve of Graph #1 there are no units. In the correct and detailed Learning Curve of Graph #3 there are units, but now in powers of 2 and powers of 10 (i.e., horizontal 2^n and vertical 10^n).

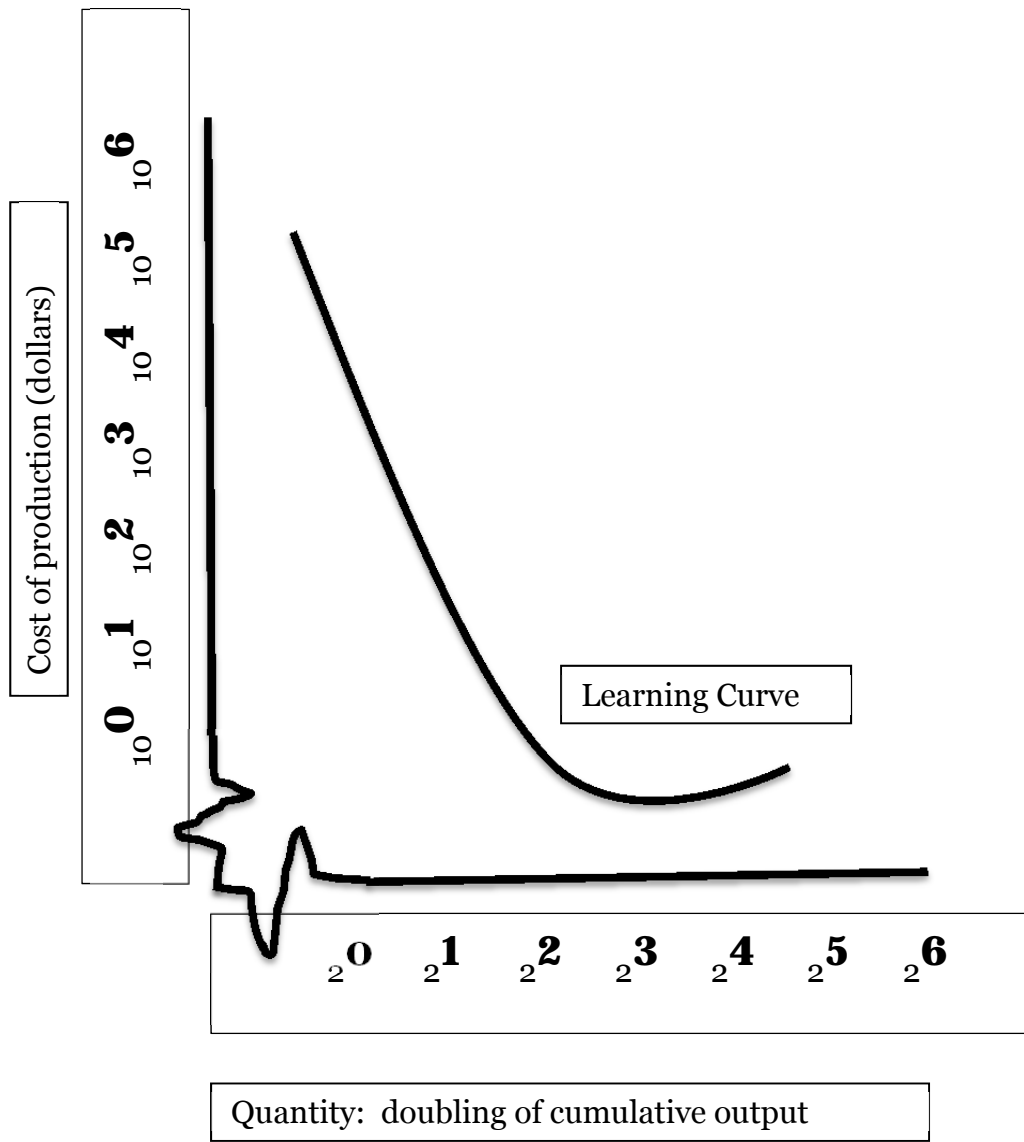
Of course, on Graph #3 you noticed yet another change in each axis. No longer is each axis portrayed as continuous. It might be no more than mere habit in the form of muscle memory when a person draws a continuous axis. It is a very frequent error. It also can be a sure sign the author of an incorrect Learning Curve has forgotten about the axis units of 2^n and 10^n .

Think about working with powers of 2 and powers of 10. Positive exponents explode values larger. Now, think about powers of less than zero. For example, 2^{-10} or 10^{-10} . How large does the power need to be before 2 raised to that negative power equals zero? It is asymptotic (i.e., rapidly approaches zero but never reaches zero).

³ The authors have seen textbooks and academic press articles with Learning Curves displayed with a horizontal axis that merely read "1, 2, 3, ... , 8, 9, 10 OUTPUT". Really?! The production process was studied from the moment in time when a production cycle yielded 1 unit of total output (e.g., one year's total output) until that moment in time when the cumulative total output of that production process' repeated cycles equaled 2^{10} (i.e., 1,024) units of cumulative total output? This is feasible (e.g., microchip manufacture), but rarely is seen in most production processes. Let's be bold and assume a 10% growth in efficiency in annual output. Year 1's total output is 1; year two's is 1.1, year three's is 1.21; etc. In what year does the cumulative total output reach 1,024? In year 49, when that year's total output is nearly 100 units.

If professionals are confused is it any wonder students are confused?

Graph #3: A LEARNING CURVE DRAWN CORRECTLY



A proposed re-function of the Learning Curve: a Knowledge Curve

As mentioned earlier, technically, individuals misspeak when referring to the Learning Curve. Let's accept that array of errors as indicative of a need for new pedagogy to assure student learning of this obviously important concept.

To assist the student's uptake of this concept let's reformulate the graph. Let's shift the focus away from cost of production and towards learning. We'll call it the Knowledge Curve. Hopefully, this will empower our students to grasp the importance of the learning on the production function and the feasible outputs. That is, learning dynamically transforms the interaction of other, static inputs.

A routine error is to graph the horizontal axis as time rather than as doublings of cumulative output. OK, let's take that as hint and make time the horizontal axis of the Knowledge Curve.

For example, an individual when attempting to learn a new version of Windows software might refer to the experience as having a steep learning curve. What the speaker typically means is that: [a] it took that person a long time to master the knowledge; or [b] the time spent learning the new operations was more costly than the reward; or [c] a great quantity of knowledge is learned quickly (at high cost or at low cost). Yes, time is in play. As noted in footnote 1, in the Learning Curve time is relevant as an indirectly implicated independent variable. Let's make the implicit explicit. Let's take common error (i.e., time instead of output) as a suggestion for reformulating the horizontal axis.

Next, let's take a hint from the word "steep".

Steep can be up or down. Steep refers to a rate of change. But, the error is not correctly identifying the thing being changed. When a confused student is thinking about the Learning Curve being steep, one problem is the student is unclear on what is changing fast. Is it total knowledge or is it total effort? In the Learning Curve the thing being changed is $\% \Delta$ Ave Cost per $2^n Q$. But it's called a learning curve. For the Knowledge Curve let's put knowledge on the vertical axis.

Steep is a relative term. We use a common reference when we put time on the horizontal axis. That will not avoid all confusion. Sometimes a student's confusion comes from confusing speed of accumulation of knowledge versus total knowledge accumulated. That is, steep can be the speed of successful absorption of one unit of knowledge or steep can be total absorption. Steep. In an accurately understood "steep Learning Curve" both concepts are key: speed and total. By making the vertical axis knowledge the visually steep slope (i.e., rise over run) conveys both speed of accumulation and total accumulation of knowledge. The Knowledge Curve that is steep has both a higher total knowledge and a rapidly rising curve.

Graph #4 is on the next page. Graph #4: Knowledge Curve: A Curve Focused on Learning as Opposed to a Curve Focused on Average Cost, we hope, avoids some of the sources of student confusion. With Graph #4 the student ought to be able to better understand Graph #2. The steep Knowledge Curve is both fast and higher. Putting time on the horizontal and knowledge on the vertical shows that steep refers to difficulty of accumulating a total quantity of knowledge. We graph learning (absorption of knowledge) against the time required to absorb said knowledge. The Knowledge Curve measures knowledge absorbed against the time required to do so.

**Graph #4: KNOWLEDGE CURVE:
A CURVE FOCUSED ON LEARNING
AS OPPOSED TO A CURVE FOCUSED ON AVERAGE COST**

