Applying Geometric Returns During Interest Rate Changes: Interest Rate Timing Volatility Variations

by

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Introduction

The United States over the last 16 years (from 1990 – 2005) has experienced some of the lowest inflation rates since the 1950's. Given these low inflation rates, it is not surprising that interest rates also have been hovering around a 16 year low. The long period of relative stability in these two financial indicators have, to some extent, lessened their influence when considering their effects.

The "out of sight; out of mind" philosophy is evident in some of today's financial planning. Ignoring the effects that interest rate changes due to inflationary pressure can have on wealth accumulation and depletion is a grievous error. The wealth differential is especially noticeable when applying an average interest rate (geometric return) which was obtained by calculated the periodic interest rates over a number of compounding periods where the rates have been consistently increasing or decreasing.

The scenarios and the examples that are provided in this paper may be considered as sudden and extreme jumps. However, these examples are used to illustrate the effects given a relatively short time period (10 years). Comparable results can be obtained even with more subtle interest rate changes given a longer time period.

The Average Rate of Return

The term "the average rate of return" is used to describe the rate of return over a given time period. There are a number of ways to calculate rates of return. Perhaps the simplest is the arithmetic mean rate of return which is obtained by taking the individual periodic rates, summing them and dividing by the number of periods. This type of calculation ignores compounding, and is considered by most individuals (with any financial training) as incorrect. To most individuals with any financial acumen the preferred method of calculating an average return over a number of compounding periods is to calculate the geometric return. The geometric mean return refers to the compounded growth over a number of compounding periods. The arithmetic mean does capture more of the volatility of the individual year's interest rates.

The geometric return is calculated by using the present value (the principle), the future value amount (the principle plus the earned interest), and the number of compounding periods. The geometric return can be obtained in a number of different ways, mathematically, use of financial tables or a financial calculator.

If a rate of return does not change over the compounding periods then the arithmetic mean rate of return is equal to the geometric mean rate of return. However if there is a change in the interest rates from compounding period to compounding period, then the geometric return must be employed to obtain the correct compounded rate of return.

For example, if an interest rate is unchanged at 7.4 % over a 10 year period then the arithmetic rate of return is 7.4% (10 X 7.4% = 74/10 = 7.4%). The geometric rate also is 7.4 $(1.074)^{10}$ = 2.04194 then $\sqrt[10]{2.04194}$ = 1.074 -1 = .074 or 7.4%), (i.e. the same as the arithmetic rate). Please see Table 1.

Table 1

Calculating the 10 year return on $1000 at a 7.41% rate of return over the entire 10 year period, compounded annually

Year	Principle	Annual Rate of Return %	Interest	Balance
0	$100,000	7.41	$7,406	$107,406
1	$107,406	7.41	$7,954	$115,360
2	$115,360	7.41	$8,544	$123,904
3	$123,904	7.41	$9,176	$133,080
4	$133,080	7.41	$9,856	$142,936
5	$142,936	7.41	$10,586	$153,522
6	$153,522	7.41	$11,370	$164,892
7	$164,892	7.41	$12,212	$177,104
8	$177,104	7.41	$13,115	$190,219
9	$190,219	7.41	$14,086	$204,304
10	$204,304			

However, if an investment was subject to a 3% interest rate for the initial five years and then to a 12 % rate for the next five years, the arithmetic rate of return would be 7.5% and the geometric rate of return would 7.405 % (employing the same mathematical principles as the example above). Please see Table 2.

Table 2

Calculating the 10 year return on $1000 at a 3% for the first 5 years an then 12% for the remainder, compounded annually.

Year	Principle	Annual Rate of Return %	Interest	Balance
0	$100,000	3.00	$3,000	$103,000
1	$103,000	3.00	$3,090	$106,090
2	$106,090	3.00	$3,183	$109,273
3	$109,273	3.00	$3,278	$112,551
4	$112,551	3.00	$3,377	$115,927
5	$115,927	12.00	$13,911	$129,839
6	$129,839	12.00	$15,581	$145,419
7	$145,419	12.00	$17,450	$162,870
8	$162,870	12.00	$19,544	$182,414
9	$182,414	12.00	$21,890	$204,304
10	$204,304			

It is interesting to note that if the interest rate sequence was reversed and the investment was subject to a 12% interest rate for the initial five years and then to a 3 % rate for the next five years, once again, using annual compounding the annual percentage return arithmetically would be 7.5% and the geometric return for the 10 year investment would be 7.405%, the same as the previous example. Please see Table 3.

Table 3

Calculating the 10 year return on $1000 at a 12% for the first 5 years an then 3% for the remainder, compounded annually.

Year	Principle	Annual Rate of Return %	Interest	Balance
0	$100,000	12.00	$12,000	$112,000
1	$112,000	12.00	$13,440	$125,440
2	$125,440	12.00	$15,053	$140,493
3	$140,493	12.00	$16,859	$157,352
4	$157,352	12.00	$18,882	$176,234
5	$176,234	3.00	$5,287	$181,521
6	$181,521	3.00	$5,446	$186,967
7	$186,967	3.00	$5,609	$192,576
8	$192,576	3.00	$5,777	$198,353
9	$198,353	3.00	$5,951	$204,304
10	$204,304			

In obtaining the geometric rate of return it does not matter which percentage is paid first (3% or 12%), $1,000 in an investment account under either scenario will have a future value of $2,043.04. and result in an average return of 7.405% (rounded to 7.41%). Tables 1, 2 and 3 give examples of compounding $1,000 under 3 different scenarios. In all three cases the future value is the same, meaning that the average rate of return must be identical in each scenario.

The geometric rate of return is the preferred method of calculating return in investment literature. Many annual reports will include a one, five and ten year average return on the company. Probably some of the most quoted calculations are from the Ibbotson and Associates. They calculate the return on the security market returns from 1929 to the present. Ibbotson gives the **geometric return** on large cap and small cap stocks, corporate and government bonds, selected T-bills and the inflation rate just to name a few measurements.

Employing Past Returns to Predict Future Cash Flows

Problems may occur when employing past returns to predict the future cash flow from an investment. The markets past behavior may be an accurate predictor of future averages, however, it is important to observe at what point specific returns will occur in the future. If calculations of the past 50 years of annual returns give an average return as 7.41%, then to blindly employ this number as the future rate of return may cause problems. For example, if a retirement consultant implies correctly that over the next 10 years that the average rate of return on an investment is 7.41% and that the investment, by using this number, may be disbursed in equal amounts over the next 10 years, there could be a problem with the cash flow.

The problem is not that the retirement consultant was correct in predicting the average future return at 7.41%. It is which pattern of individual annual returns will generate the average annual return of 7.41%. If the returns are consistent for the next ten years (i.e., 7.41% each year), no problem, the investment will payout in equal amounts as annuitized over the 10 year period by employing 7.41% in the annuity calculation as the future interest rate. However if the retirement consultant implies that an investment will earn a constant annual rate of 7.41% over the next 10 years but the average is comprised of a combination of some years when the annual rates are lower than average combined with some years when the annual rates are higher than average, then the investment payout may not support the 10 years as annuitized. For example, consider the following 3 scenarios.

Scenario 1: An investment of $100,000 is placed in an account that pays a guaranteed interest rate of 7.405% annually. This interest rate will allow the recipient to withdraw $14,505.50 (an annuity) per year for each of the 10 years. At that time the ending balance in the account would be 0. The variable to note is the interest rate. It stays constant at 7.405% annually for the entire 10 year period. Please see Table 4.

Table 4

Scenario 1: $100,000, spending $14,505 each year given the derived annual percentage return for both Scenario 1 and 2, **7.405%**

Year	Principle	Interest Rate %	Annual Interest	Principle & Interest	End of Year Spending
0	$100,000	7.41	7,410	$107,410	-$14,506
1	$92,905	7.41	6,884	$99,789	-$14,506
2	$85,283	7.41	6,319	$91,603	-$14,506
3	$77,097	7.41	5,713	$82,810	-$14,506
4	$68,305	7.41	5,061	$73,366	-$14,506
5	$58,860	7.41	4,362	$63,222	-$14,506
6	$48,717	7.41	3,610	$52,326	-$14,506
7	$37,821	7.41	2,803	$40,623	-$14,506
8	$26,118	7.41	1,935	$28,053	-$14,506
9	$13,548	7.41	1,004	$14,552	-$14,506
10	$0		0	$0	$0

Scenario 2: The investment of $100,000 is placed in an account that pays 12% for each of the initial 5 years and 3% for each of the last 5 years. The interest rate can be calculated by deriving the total interest factor over this period. Years 1 – 5 (at 12%) has an interest factor of 1.762342 and years 6-10 (at 3%) has an interest factor of 1.159274. In other words if $100,000 were deposited into an account that had this rate pattern (12% then 3%) the total accumulation at the end of the 10 years, if no withdrawals were made, would be $204,103.70. If both interest factors are added together (2.041037), and the overall (10 year) interest rate extrapolated employing tables or a calculator, the rate would be 7.41%, the same as before. However, in scenario 2, with years 1 through 5 at 12% and years 6 through 10 at 3%, the balance remaining in the account (principle) at the end of 10 years would be $20,462.11. Please see Table 5.

Table 5

Scenario 2: $100,000 invested, spending $14,505.61 each year based on a 7.41 % average annual return derived by taking a 12% rate of return for initial 5 years and 3% rate of return for the subsequent 5 years

Year	Principle	Interest Rate %	Annual Interest	Principle & Interest	End of Year Spending
0	100,000	12.00	12,000	112,000	-$14,506
1	97,494	12.00	11,699	109,194	-$14,506
2	94,688	12.00	11,363	106,051	-$14,506
3	91,545	12.00	10,985	102,530	-$14,506
4	88,025	12.00	10,563	98,588	-$14,506
5	84,082	3.00	2,522	86,605	-$14,506
6	72,099	3.00	2,163	74,262	-$14,506
7	59,756	3.00	1,793	61,549	-$14,506
8	47,044	3.00	1,411	48,455	-$14,506
9	33,949	3.00	1,018	34,968	-$14,506
10	20,462		$0	$0	$0

<u>Scenario 3</u>: The $100,000 investment is placed in an account that pays 3% for the each of the initial 5 years and 12% for each of the last 5 years. Once again, the interest rate can be calculated by deriving the total interest factor over this period. Years 1 through 5 (at 3%) has an interest factor of 1.159274 and years 6 through 10 (at 12%) has an interest factor of 1.762342. Once again, if $100,000 were deposited into an account that had this rate pattern (3% then 12%) the total that would accumulate, if no withdrawals were made, at the end of the 10 years would be an identical $204,103.70. As in the previous scenario, if both interest factors are added together (2.041073), and the overall interest rate were extrapolated, the rate would be the same 7.41%. In Scenario 3, with years 1 through 5 at 3% and years 6 through 10 at 12%, the balance remaining in the account at the end of 10 years would be a *negative* $23,570.10. Please see Table 6.

<u>Table 6</u>

<u>Scenario 3</u>: $100,000 invested, spending $14,505.61 each year based on a 7.41 % average annual return, derived by taking a 3% rate of return for the initial 5 years and 12% rate of return for the subsequent 5 years

Year	Principle	Interest Rate %	$ Annual Interest	Principle & Interest	End of Year Spending
0	$100,000	3.00	$3,000	$103,000	-$14,506
1	$88,494	3.00	$2,655	$91,149	-$14,506
2	$76,644	3.00	$2,299	$78,943	-$14,506
3	$64,437	3.00	$1,933	$66,370	-$14,506
4	$51,865	3.00	$1,556	$53,421	-$14,506
5	$38,915	12.00	$4,670	$43,585	-$14,506
6	$29,079	12.00	$3,490	$32,569	-$14,506
7	$18,063	12.00	$2,168	$20,231	-$14,506
8	$5,725	12.00	$687	$6,412	-$14,506
9	-$8,093	12.00	-$971	-$9,065	-$14,506
10	-$23,570		$0	$0	$0

Note that in all three scenarios the overall interest rate for the time period was derived at 7.41%, just the timing of when the rates occurred was changed. The implication of these timing changes for anyone involved in planning future expenditures is to be careful when assuming the volatility of future interest rate movements. If the economy appears to be coming out of a period of low rates of return and heading into a period of high rates of return (as the U.S. economy appears to be presently doing) the results will be substantially different then if interest rates are trending in the opposite direction.