Managerial Economic enrollees:

In last week's lecture on Chapter 4 I fell victim to poor eyesight. As I am sure all students that have had me for law can tell you, the hardest thing to see is the thing that is not there. In last week's lecture I failed to alert you to an important and missing equation. This failure becomes obvious when you attempt to answer question 4:133:10.

4:133:10 asks:

"Suppose that Milton has $50 to be divided between corn and beans and that the price of beans is $.50 per pound. What will be the relationship between the price of corn and the amount of corn he will buy if $U = \log Qc + 4 \log Qb$, where U is the utility, Qc is the quantity of corn he consumes (in pounds), and Qb is the quantity of beans he consumes (in pounds)?"

That the utility function, U, is in logarithms means that the units of measure are constant in terms of percentage, rather than constant in terms of pounds. That is, an ordinary function would count 1 pound, 2 pounds, 3 pounds; while this log function counts 1 percent increase, 2 percent increase, 3 percent increase. Otherwise, it is just another linear function.

The key to answering any question is finding the right equation or the right set of equations. The key to answering question 4:133:10 is an equation we spent a considerable amount of time on: page 122, Eq. (4.7)

$MRS = Pc/Pf$.

Additionally, we spent a considerable amount of time on the extensions of that equation in the text on page 123, coming up with

If $MRS > Pc/Pf$ then add more Clothing

and

If $MRS < Pc/Pf$ then add more Food.

Since you are unlikely to recall the food and clothing example down to the detail of which axis was used to display food and which axis was used to clothing, let us now take a moment to establish a convention for how to assign which goods to which axis. This will also allow us to generalize the equation $MRS = Pc/Pf$ into a more easily remembered form. For our purposes let's agree that the first good we encounter will always be assigned to the Y axis of a utility function and the second good will be assigned to the X axis of a utility function (and its related budget constraint). Therefore, the generalized form of Eq. (4.7) is

$MRS = Px/Py$

and the generalized forms of the extended equation are

If $MRS > Px/Py$ then add more of X (if MRS above, add to top)

and

If $MRS < Px/Py$ then add more of Y (if MRS below, add to bottom).

In fact, the assignment of axes is arbitrary and has no impact on the calculation of the quantities of X and Y. The assignment of axes does set the magnitude of the MRS as either an integer or as that integer's inverse, and similarly sets the sequence in which you express the exchange of one good for the other (e.g., food for clothing versus clothing for food).

Our problem here is that nowhere in the text (to date) have you been shown an equation for the calculation of the MRS. On page 115 the text gives the definition of the MRS, the Marginal Rate of Substitution. The MRS is -1 times the slope of the indifference curve. The utility curve is the indifference curve. The MRS is the rate of exchange, repeat exchange, of one good for another good while holding constant the utility. For question 4:133:10 the MRS, the -1 times the slope of U, is -1 times the Change in corn/Change in beans: that is, the rate of exchange. But where is the means of calculating the slope? The partial derivative of corn gives the rate of change in U give a change in corn; it does not give the rate of exchange.

Let's go back to the beginning. What is the simplest equation for a slope? Rise/Run, or (Change in Y) / (Change in X). From that we went to derivatives. Recall page 41 and Eq. (2.3) that defined (Change in Y) / (Change in X) as $\Delta Y/\Delta X$ and also recall page 42 and Eq. (2.5) that defined dY/dX as $\Delta Y/\Delta X$ approaching the limit of zero for ΔX. Our problem here is how to calculate rise/run of an --indifference curve--. What we want is dQc/dQb because $U = \log Qc + 4 \log Qb$ and by our convention we will assign corn to the Y-axis. Our problem is that so far in the text you only have taken partial derivatives of the type $\delta U/\delta Qc$, but here that appears to be of no use.

Recall that an individual indifference curve holds utility constant. To change the level of utility it is necessary to move to another indifference curve Look at page 122, Fig. 4.4. Figure 4.4 shows one budget constraint line and three indifference curves. Each indifference curve has a different utility. Are these three indifference curves parallel? If three curves are parallel, then the slope of one curve is equal to the slope of the other two curves. If, however, three curves are not parallel, then each curve has its own slope. Indifference curves need not be parallel.

Note how indifference curves are unlike the page 118 budget constraints in Fig. 4.5 when the Px/Py price ratio is held constant, but indifference curves are like the budget constraint in Fig. 4.6 when the Px/Py price ratio changes. This will be a hint for finding the solution to our current problem.

Intuitively, when you look at the utility function $U = \log Qc + 4 \log Qb$ you just "know" that the MRS is 4/1 (or 1/4, depending upon the axis assignment convention), because you will exchange one corn for four beans. The difficulty is proving this by math. To learn how to prove it by math turn to text pages 228 and 229. You would be wise to make a note on page 115 and on page 122 to go see pages 228 and 229, and a note on 228 to go see 115 and 122. (If your math skills are weak, then on page 228 also put a note to go see pages 41 and 42).

In prior chapters we have learned how to find the slope of a function. It is rise/run, or the first derivative of a function with one variable. Pages 228 and 229 explain how to find the slope of a function with two variables by using the first partial derivatives.

In class I said that the MRS was merely the first time we would run into this concept. The most important version of this concept is the Marginal Rate of Technical Substitution, or MRTS, found on pages 228 and 229. An "isoquant" is mathematically similar to an indifference curve in that both hold constant something: an indifference curve holds constant utility, while an isoquant holds constant output. (Note, on page 231 you will once again encounter the price ratios.)

As you read pages 228 and 229 pay critical attention to the subscripts as the variables will jump in unexpected ways. On page 229 note that the MP1 and the MP2 are marginal products and thus are the partial derivatives (which you know how to take) of the isoquant with respect to X1 and X2. Lastly, also note that there is a typographical error on page 229. In the equation immediately above Eq. (7.8) the first X1 is missing a δ, thus the far left element of the equation immediately above Eq. (7.8) should be $\delta Q/\delta X1$ instead of $\delta Q/X1$.

Recall page 41 and Eq. (2.3) that defined (Change in Y) / (Change in X) as $\Delta Y/\Delta X$ and also recall page 42 and Eq. (2.5) that defined dY/dX as $\Delta Y/\Delta X$ approaching the limit of zero for ΔX. You do NOT break apart the symbol dY/dX. However, if you recall pages 41 and 42 you should have no difficulty in seeing how to maneuver algebraically and thus to create the object of our desire, the slope of a function with two variables, dX1/dX2 (which under our axis assignment convention is dY/dX).

Page 229 Eq. (7.8) gives the slope of a two-function curve in three forms.

Note how the subscripts move around. I suggest you actually work from the equation immediately above (7.8) to Eq. (7.8).

For our purposes in question 4:133:10 we need the form in the middle or the form on the right. Recall the convention of assigning the first variable to the Y-axis and the second variable to the X-axis. The middle term is -1 times the rate of change in Y caused by the rate of change in X. In contrast, however, the left term reads (in effect) dX/dY.

For question 4:133:10 the MRS is $(-1) * \{[(-)(\delta U/\delta Qc)] / (\delta U/Qb)\}$. Reread that equation and think about each of it components. Thus,

(-1)*{[(-)(1)]/(4)} = 1/4.
Note that the minus on the slope is forced by the algebraic manipulation of the equations leading to page 229 Eq. (7.8) rather than the sign on the coefficients of the question 4:133:10 utility function.

Now, let's solve 4:133:10. From the text of question 4:133:10 we have the budget constrain
I = $50;
and we have Pb = $0.50;
and we have U = log Qc + 4 log Qb.
From above we have MRS = 1/4.
Using the equation
MRS = Px/Py,
where Pc is Py and Pb is Px, we therefore can solve for Pc via
1/4 = $0.50/Pc;
or
Pc = $2.00.
So, what is the relationship between Pc and Qc?
We will consume one-fourth as many pounds of corn as we do beans, and we will pay four times as much per pound for corn as we do for beans.

Can we find Qc and Qb with the information that is given?

To find Qc and Qb is to know four variables: Qc, Pc, Qb, and Pb. We started off knowing only one those these four variables (i.e., Pb = $0.50), and we are given two equations. By way of the MRS we were able to add a third equation and uncover the second price variable (i.e., Pc = $2.00). Thus, we now have two knows and two unknowns with two equations, which --might-- be doable, depending upon the structure of the functions with which we must work.

Recall the "math trick" from the first exam for finding the equilibrium price and quantity: --at-- equilibrium items with different subscripts become identical. Since
I = $50 and since Pb = $0.50 and since Pc = $2
50 = 0.5 Qb + 2 Qc.
Also recall that U = I --at-- the Equilibrium (read maximum) Market Basket. This next step is critical: what value do we use for when U = I?. The question becomes, --may-- we set U and I equal to zero? Is the utility equal to zero? No, utility is equal to some number greater than zero. Can we set the utility equal to 50? Yes, we can set U = 50 since utils have an arbitrary scale. This is a second missing "math trick". When we set U = I = 50, then we have
I = 50 = 0.5 Qb + 2 Qc
and we have
U = 50 = 4 Qb + Qc.
If we subtract U from I, then we get

3.5 Qb = Qc
substituting 3.5 Qb into I for Qc we get
Qb* = 6.67 (with a Pb* = $0.50 since Pb always equals $0.50)
substituting Qb* into I yields
Qc* = 23.3325 (with a Pc* = $2.00)

I am sorry I failed to alert you to this missing equation in the text. Your thanks should go out to a cadre of seven students who stayed after class on Thursday for over a half-hour in a search for a solution to this "simple" question.

To sum up, to solve this type of question you need to recall that
MRS = Px/Py
MRS is (-1) * {[(-)(δU/δQy)] / (δU/δQx)}
and
do NOT set U = I = 0, but rather set equal to the value given for I.
With those three "math tricks" these types of questions are really quite simple. However, the text and your instructor failed you in not providing two of the three needed tricks. The third trick, arguably, could have been deduced by an educated guess. However, the groundwork for that educated guess was not set up without the second trick.