

*A Steep Learning Curve*

<http://cba.unomaha.edu/faculty/mohara/web/NEBA-2011-Steep-Learning-Curve.pdf>

by

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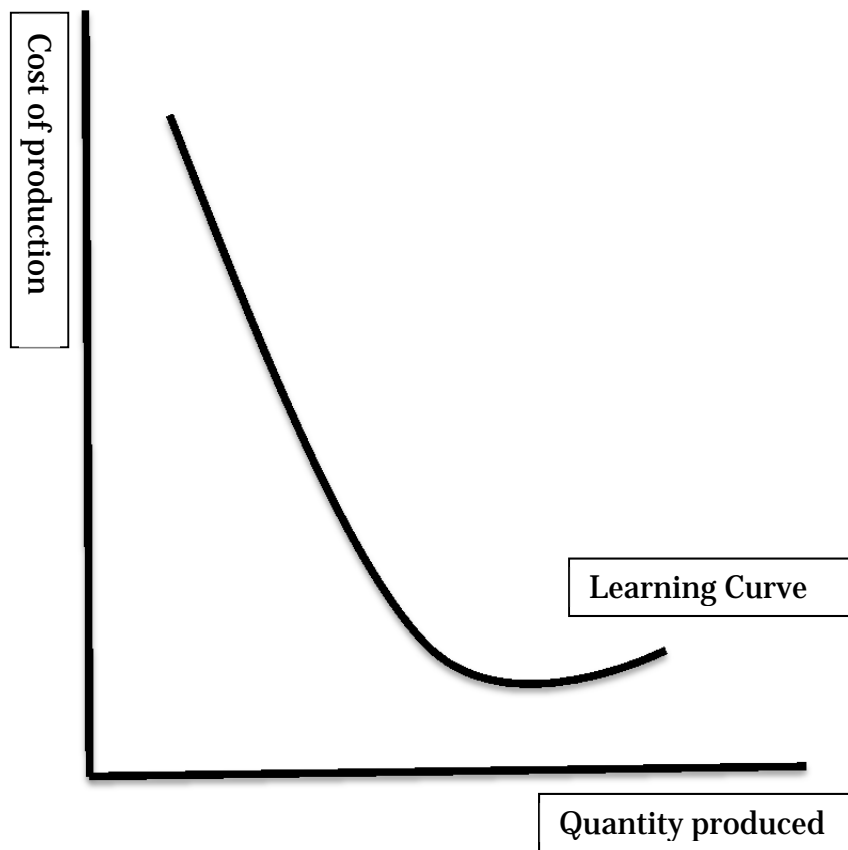
presented to

Nebraska Economics and Business Association

Norfolk, Nebraska, October 2011

**ABSTRACT:** Even inattentive business students have heard the phrase "a steep learning curve". However, rarely is that phrase used correctly. In the vernacular "a steep learning curve" means little more than a lot of learning in short period of time. Technically, the relationship of learning curve is far more precise. This pedagogy manuscript will discuss: first, contrasting how not to as well as how to draw a learning curve; second, some of the mathematical attributes of the learning curve graph and its uses; and third, a few glaring errors routinely made by even knowledgeable persons when drawing a learning curve.

**A LEARNING CURVE THAT AVOIDS CLEAR ERROR,  
BUT INVITES STUDENT CONFUSION**



Let's recall Adam Smith's identification of three sources of gains from specializing labor inputs via the division of labor. To wit: [i] differences in natural ability; [ii] avoidance of time expenditures as shift from one task to another task; and, of interest to us here, [iv] learning. Repetitive production creates the opportunity for labor and the opportunity for management to learn more efficient production techniques.

The learning curve seeks to portray changes in the average cost of production with increases in the cumulative quantity of production. More specifically, the focus is on percentage changes in the average cost of production; and, the focus is on doublings of cumulative output.

The graph with title "A Learning Curve that Avoids Clear Error, but Invites Student Confusion" is accurate because of what it omits. It invites student error because what it omits is not what ordinarily is found on a graph. The horizontal axis is true as far as it goes. So too the vertical axis: true as far as it goes. The problem is what is missing is not as ordinarily would be expected from a graph. Saying nothing avoids saying something wrong.

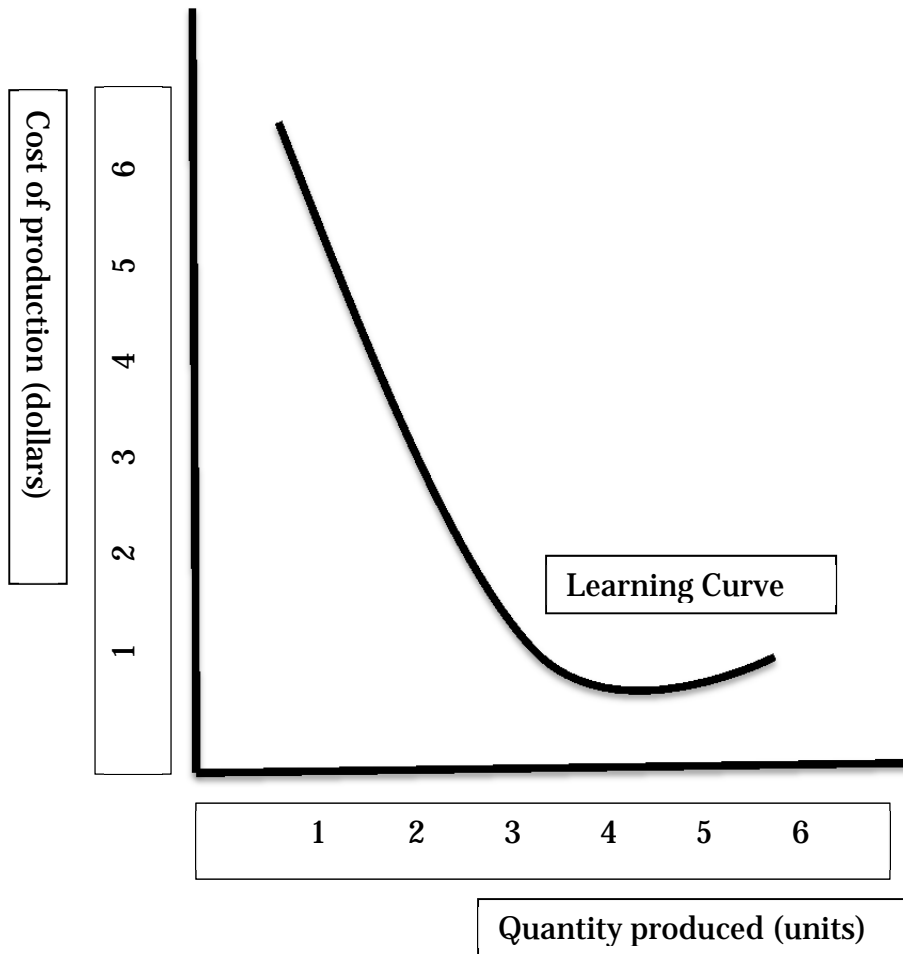
On a learning curve the horizontal axis is a specific measure of quantity produced. The ordinary measure of quantity has a horizontal axis count off increments of units of quantity. For example, an ordinary quantity axis would read 1 unit, 2 units, 3 units, etc. Not so the learning curve horizontal axis. For a learning curve the metric for quantity is a doubling of output. That is, the horizontal axis of a learning curve does count off 1, 2, 3, 4; but, those are doublings. Or, more mathematically, powers of 2. The learning curve horizontal axis counts off  $2^0$ ,  $2^1$ ,  $2^2$ ,  $2^3$ ,  $2^4$ , etc. Recall that any number raised to the zero power is 1 and any number raised to the power of 1 is itself.

For example, off  $2^0$  equals one and  $2^1$  equals two. That is, on the learning curve the horizontal axis, in units, counts off as 1, 2, 4, 8, 16, etc.

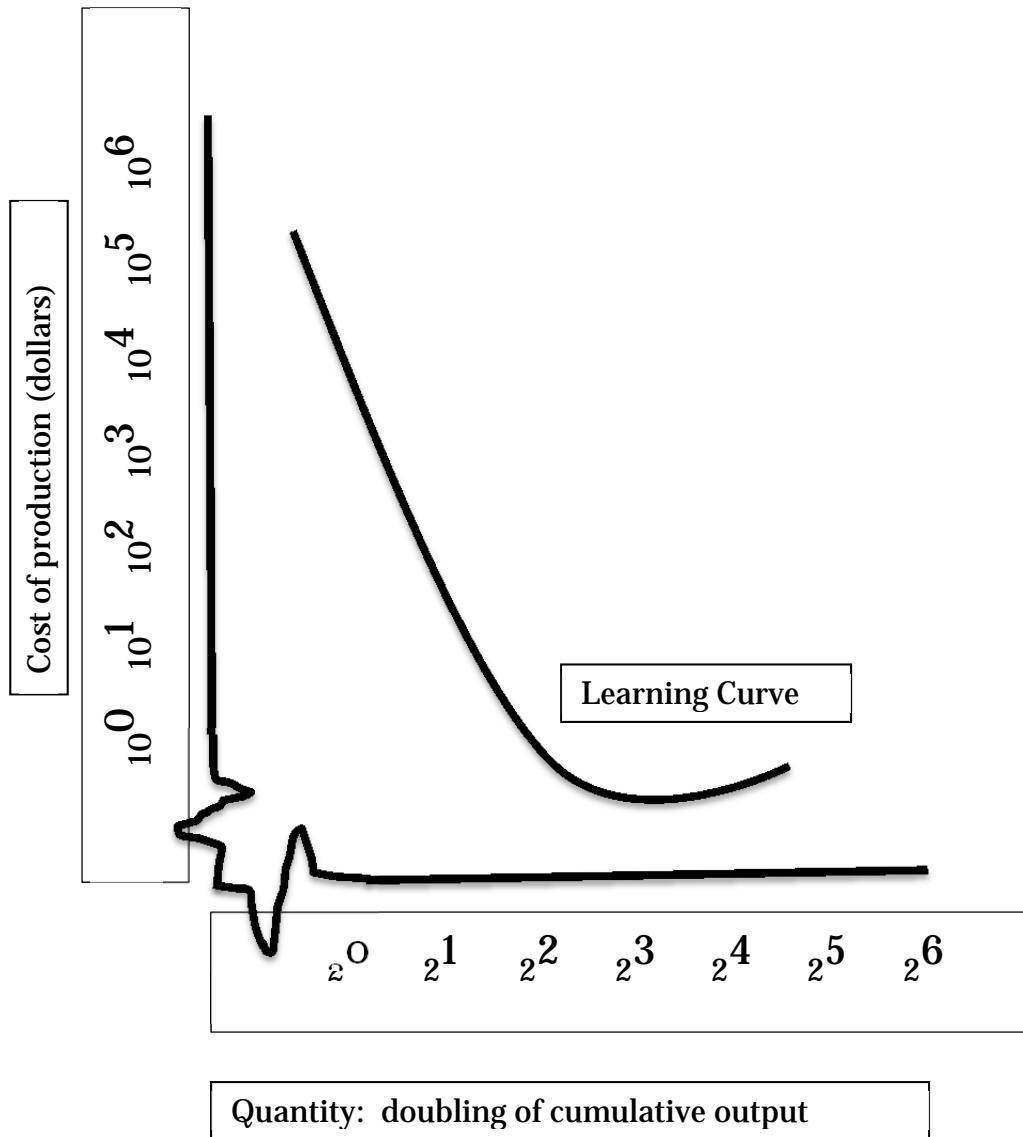
The vertical axis is similar to but materially different than the horizontal axis.

The learning curve seeks to explain changes in average cost of production. Specifically, *percentage changes* in the average cost of production. Accordingly, the vertical axis of the learning curve also counts off in powers, but this time in powers of 10. That is, the learning curve axis does not count off \$1, \$2, \$3. Instead, it counts off  $10^0$ ,  $10^1$ ,  $10^2$ ,  $10^3$ ,  $10^4$ ; or, if you prefer, \$1, \$10, \$100, \$1,000, and \$10,000. Using base 10 raised to a power means that *equal vertical distances* on the axis measure out *equal percentage changes*.

**A LEARNING CURVE AS ROUTINELY DONE: that is, DONE WRONG**



## A LEARNING CURVE DONE CORRECTLY



Of course, you noticed yet another change in each axis. Each axis no longer is portrayed as continuous. Think about working with powers of 2 and about working with powers of 10. Now, think about powers less than zero. For example,  $2^{-10}$  or  $10^{-10}$ . How large does the power need to be before 2 raised to that negative power equals zero?