IDENTIFYING REGIME CHANGES IN MARKET VOLATILITY

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Abstract

A casual inspection of a graph of volatility indexes over time indicates that volatility has undergone infrequent, but significant, shifts in its average level. The purpose of this article is to test for multiple structural breaks in the mean level of market volatility measured by the VIX and VXO and to identify statistically the dates of these mean shifts. We find evidence of three distinct periods. The periods correspond to a pre-1992 period, a 1992-1997 period, and a post-1997 period. We find that the mean volatility, as well as its standard deviation, was lowest during the 1992-1997 period. Our findings provide statistical evidence consistent with popular beliefs that market volatility changes over time.

JEL classifications: G10, G12

I. Introduction

Since its introduction by the Chicago Board Options Exchange (CBOE) in 1993, the volatility index (symbol VIX) has been a popular indicator of short-term market volatility. Traders use up-to-the-minute estimates of expected volatility to gauge investor fear (Whaley 2000). Copeland and Copeland (1999) examine the VIX as a signal for market timing for moving between two portfolio strategies such as style (value vs. growth) and size (large vs. small cap). Hyerczyk (2001) finds that the use of the VIX enhanced the performance of trading signals, whereas, Corrado and Miller (2003) investigate the predictive powers of VIX.

When the VIX was first introduced, the index was calculated based on volatilities implied in options on the S&P 100 index (symbol OEX). Two significant changes were made to the index in September 2003. First, the VIX was changed so as to be based on options on the S&P 500 index (symbol SPX). The OEX-based index was renamed the VXO index. Second, the method used in the VIX index calculation was altered to reflect a much larger range of options across strike prices. Given that the S&P 500 is often perceived as a proxy for the total market index, the changes make the VIX a viable tool for investors to hedge market volatility and paved the way for the introduction of futures on the VIX in March 2004. Futures on the VIX was predicted to be "one of the most popular instruments" at the CBOE.[1]

Recognizing the investors' need for option contracts on market volatility for portfolio insurance purposes, as advocated in Whaley (1993), the Chicago Futures Exchange further plans to list options on the VIX futures contracts in the near future, pending regulatory approval.

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[1] Mehta, N. 2004, Product Profile: Futures on the VIX, *Futures Industry Magazine* March / April 2004 Issue (p. 68-70).

Inspection of a graph of the VIX and VXO indexes over time indicates that volatility has undergone infrequent, but significant, shifts in its average level. Mean reversion is a well-known property of market volatility and it is often an underlying assumption in financial asset pricing theories. Mean reversion essentially suggests that when volatility reaches high levels, it is likely to come down, and when it is low, it will likely rise. Over time volatility will indeed revert to a mean; however, it appears that the mean to which volatility reverts is unlikely to be constant through time. Our purpose is to investigate statistically whether there have been any structural changes in the mean levels of the VIX (based on SPX) and VXO (based on OEX) and to identify statistically the dates of these mean shifts along with confidence bands around these dates. Given that the VIX and VXO are widely used as proxies for market volatility, the research is of significant importance to practitioners, as numerous investors build investment and hedging strategies based on their understanding of the mean levels of the indexes. This article will also provide information to academia on future studies of the pricing and characteristics of VIX futures and options written on VIX futures. The importance of accounting for the dynamic nature of market risk is recently pointed out by Mayfield (2004). He models the evolution of volatility as Markov processes with a high-volatility state followed by a low-volatility state. In his model the market risk premium is modeled as a function of the process generating the volatility states. His results suggest that the simple historical average of excess market volatility obscures significant variation in the market risk premium and that half of the measured risk premium is associated with the risk of future changes in the level of market volatility.

This article contributes to the literature of market volatility studies. Given that change in market volatility signal changes in market participants' perception of risk in stock price movements, market volatility is researched extensively. Although many researchers observe that aggregate market volatility changes over time (Schwert 1989, 1998), recent work by Campbell et al. (2001) find that idiosyncratic (firm-level) risk has increased substantially over time but that aggregate market volatility has remained stable from 1926 to 1997. The volatility measures in prior studies, whether they are simply based on standard deviation (Schwert 1998), or some modified forms of standard deviation (French, Schwert, Stambaugh 1987) are essentially backward looking based on realized volatility. The VIX and VXO are option-implied volatilities; hence, they provide us an opportunity to look at market volatility from a forward-looking perspective.

An additional motivation for our research relates to mean shifts in volatility. Granger and Ding (1996) find evidence of long-memory in absolute stock returns. More recent work by Lobato and Savin (1998), Diebold and Inoue (2001), and Granger and Hyung (2004) present evidence that the finding of long-memory may be the result of neglected occasional structural breaks in the series. Although it is beyond the scope of this article to provide a definitive answer about which is the more appropriate characterization of the data, our results offer an alternative picture to a long-memory process. A similar argument applies to the often-found feature of integrated GARCH processes in volatility.

We use the Bai and Perron (1998, 2003a,b, 2004) method to test for multiple structural breaks in the mean levels of the VIX and VXO volatility indexes for the 1990-2003 period. In Monte Carlo experiments, Bai and Perron (2004) find that the Bai and Perron method is powerful in detecting structural breaks. We find evidence of three distinct periods in the VIX and VXO indexes: a pre-1992 period, a 1992-1997 period, and a post-1997 period. We find that the mean volatility (as well as its standard deviation) was lowest during the 1992-1997 period. Our findings provide statistical evidence consistent with popular beliefs that market volatility changes over time.[2] Based on the notion that market volatility changes over time, a stream of research attempts to capture the time variation in volatility by looking at changes in macroeconomic and business factors as well as expected stock returns to explain the change (e.g. Bakshi and Chen 1996, Hamilton and Lin 1996, Bittlingmayer 1998). In particular, Schwert (1998) relates volatility changes to the business cycle as well as economic variables, such as inflation, industrial production, and debt levels in the corporate sector. Campbell et al. (2001) show that stock market volatility helps to predict Gross Domestic Product (GDP) growth. The scope of our paper, however, is limited to documenting the structural breaks in the aggregate level of volatility.

II. Econometric Procedure

We use the Bai and Perron (1998, 2003a,b, 2004) structural break method to test for infrequent structural breaks in market volatility indexes.[3] In Monte Carlo experiments, Bai and Perron (2004) find that the Bai and Perron (1998) method is quite powerful in detecting structural breaks. We apply the Bai and Perron (1998) method to the VIX and VXO indexes and find evidence of multiple structural breaks in the mean level of these series. Following Bai and Perron (2003a), we regress the volatility indexes on a constant and test for structural breaks in the constant. Consider such a regression model with m breaks ($m+1$ regimes),

$$v_t = \beta_j + \varepsilon_t, t = T_{j-1} + 1, \ldots, T_j, \tag{1}$$

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[2] In a recent paper Eizaguirre, Biscarri, and de Gracia Holdalgo (2004) identify endogenous breakpoints in realized volatility obtained from a return-GARCH model for Spain over the 1941 to 2001 period. They employ the Bai (1997) sequential breakpoint procedure and find one structural change in realized volatility in 1972. We use the more powerful nonsequential procedure of Bai and Perron (1998, 2003a,b, 2004) and use forward looking implied volatility.

[3] This section draws heavily from Rapach and Wohar (2005).

for $j=1,...,m+1$, where v_t is a volatility index in period t and β_j ($j=1,...,m+1$) is the mean level of volatility index in the jth regime. The m-partition $(T_1,...,T_m)$ represents the breakpoints for the different regimes (by convention, $T_0=0$ and $T_{m+1}=T$.) Bai and Perron (1998, 2003a) explicitly treat these breakpoints as unknown, and estimates of the breakpoints are generated using the least squares principle. Consider estimating equation (1) via least squares. For each m-partition $(T_1,...,T_m)$, the least squares estimates of β_j are generated by minimizing the sum of squared residuals,

$$S_T(T_1,...,T_m) = \sum_{i=1}^{m+1} \sum_{t=T_{i-1}+1}^{T_i} (v_t - \beta_i)^2 . \qquad (2)$$

Let the regression coefficient estimates based on a given m-partition $(T_1,...,T_m)$ be denoted by $\hat{\beta}(\{T_1,...,T_m\})$, where $\beta=(\beta_1,...,\beta_{m+1})'$. Substituting these into equation (2), the estimated breakpoints are given by

$$(\hat{T}_1,...,\hat{T}_m) = \arg\min_{T_1,...,T_M} S_T(T_1,...,T_m), \qquad (3)$$

where the set of admissable m-partitions is subject to a set of restrictions given below. It is clear from equation (3) that the breakpoint estimators correspond to the global minimum of the sum of squared residuals objective function. With the breakpoint estimates in hand, it is straightforward to calculate the corresponding least-squares regression parameter estimates as $\hat{\beta}=\hat{\beta}(\{\hat{T}_1,...,\hat{T}_m\})$. Bai and Perron (2004) develop an efficient algorithm for the minimization problem in equation (3) based on the principle of dynamic programming.

Bai and Perron (1998) consider testing procedures aimed at identifying the number of structural breaks (m) in equation (1). They begin by specifying a statistic for testing the null hypothesis of no structural breaks against the alternative that there are $m=b$ breaks. Let

$(T_1,...,T_b)$ be a partition such that $T_i = [T\lambda_i]$ ($i = 1,...,b$). Also define R such that $(R\beta)' = (\beta_1 - \beta_2,...,\beta_b - \beta_{b+1})$. Bai and Perron (1998, 2003a) specify the following statistic:

$$F_T(\lambda_1,...,\lambda_b) = \frac{1}{T}\left(\frac{T-(b+1)2}{2b}\right)\hat{\beta}' R'[R\hat{V}(\hat{\beta})R']^{-1} R\hat{\beta}\ , \tag{4}$$

where $\beta = (\beta_1,...,\beta_{b+1})'$ is the vector of regression coefficient estimates, and $\hat{V}(\hat{\beta})$ is an estimate of the variance-covariance matrix for $\hat{\beta}$ that is robust to heteroskedastic and serial correlation. Bai and Perron (1998, 2003a) next consider a type of maximum F-statistic corresponding to equation (4),

$$SupF_T(b) = F_T(\hat{\lambda}_1,...,\hat{\lambda}_b)\ , \tag{5}$$

where $\hat{\lambda}_1,...,\hat{\lambda}_b$ minimize the global sum of squared residuals, $S_T(T\lambda_1,...,T\lambda_b)$, under the restriction that $(\hat{\lambda}_1,...,\hat{\lambda}_b) \in \Lambda_\pi$, where $\Lambda_\pi = \{(\lambda_1,...,\lambda_b); |\lambda_{i+1} - \lambda_i| \geq \pi, \lambda_1 \geq \pi, \lambda_b \leq 1-\pi\}$ for some arbitrary positive number π (the trimming parameter). Bai and Perron (1998, 2003a) develop two statistics, what they call the “double maximum” statistics, for testing the null hypothesis of no structural breaks against the alternative hypothesis of an unknown number of breaks given an upper bound M. The first double maximum statistic is given by

$$\text{UDmax} = \max_{1 \leq m \leq M} SupF_T(m)\ . \tag{6}$$

The second double maximum statistic, WDmax, applies different weights to the individual $SupF_T(m)$ statistics so that the marginal p-values are equal across values of m; see Bai and Perron (1998, p. 59) for details. Finally, Bai and Perron (1998, 2003a) specify what they label the $SupF_T(l+1|l)$ statistic to test the null hypothesis of l breaks against the alternative hypothesis of $l+1$ breaks. It begins with the global minimized sum of squared residuals for a model with l breaks. Each of the intervals defined by the l breaks is then analyzed for an additional structural break. From all of the intervals, the partition allowing for an additional break that results in the largest reduction in the sum of squared residuals is treated as the model with $l+1$ breaks. The $SupF_T(l+1|l)$ statistic is used to test whether the additional break leads to a significant reduction in the sum of squared residuals. Bai and Perron (1998, 2003a) derive asymptotic distributions for the double maximum and $SupF_T(l+1|l)$ statistics and provide critical values for various values of π and M.

A nice feature of the Bai and Perron (1998, 2003a) method is that it allows for quite general specifications when computing test statistics and confidence intervals for the break dates and regression coefficients. These specifications include autocorrelation and heteroskedasticity in the regression model residuals, as well as different moment matrices for the regressors in the different regimes. We use the most general Bai and Perron (1998, 2003a) specification that allows for all of these features, so that using the notation of Bai and Perron (2004), we set cor_u = 1 and het_u = 1 in our applications below. These conditions are quite general when no lagged dependent variables are included in equation (1) above. These conditions are sufficient to capture the most common features of volatility. For example, it is widely believed that returns exhibit conditional heteroscedasticity and hence, volatility should exhibit serial correlation. This is accommodated in the set up described above and need not be modeled explicitly.

Bai and Perron (1998) discuss a sequential application of the $SupF_T(l+1|l)$ statistics—a specific-to-general modeling strategy—as a way to determine the number of structural breaks. While Bai and Perron (2004) find that this procedure performs well in some settings, its performance can be improved upon when multiple breaks are present, as the $SupF_T(1|0)$ statistic can have low power in the presence of multiple breaks. On the basis of extensive Monte Carlo simulations, Bai and Perron (2004) recommend the following strategy to identify the number of breaks. First, examine the double maximum statistics to determine if any structural breaks are present. If the double maximum statistics are significant, then examine the $SupF_T(l+1|l)$ statistics to decide on the number of breaks, choosing the $SupF_T(l+1|l)$ statistic that rejects for the largest value of *l*. Finally, Bai and Perron (2004) recommend using a trimming parameter of at least 0.15 (corresponding to $M=5$) when allowing for heteroskedasticity and serial correlation, and we follow this recommendation in our applications.[4]

III. Data Description

The VXO and VIX series are obtained from the Chicago Board Options Exchange. When the VIX was first introduced, the index was calculated based on volatilities implied in options on the S&P 100 index (OEX) and it was calculated based on the Black-Scholes option pricing model. It was a weighted average of option implied volatilities of eight at- or nearby-the-money call and put OEX options with average time to maturity of 30 days. In September 2003, the OEX-based index was renamed the VXO index and a new volatility index based on options on the S&P 500 index (SPX) was created assuming the symbol of VIX. The method used in the new VIX index calculation was altered to reflect a much larger range of options across strike prices and it does not rely on any option pricing model. It estimates expected market volatility by averaging the weighted prices of both calls and puts across a range of strikes[5]. CBOE further created an identical historical record for the new VIX dating back to 1986 and continues to provide the S&P 100 index based VXO index so investors can compare the two indexes side-by-side. The VXO series available from the COBE begins from January 2, 1986 and the VIX series

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[4] We implement the Bai and Perron (1998, 2003a,b) method using the GAUSS program available from Pierre Perron's home page at http://econ.bu.edu/perron/.

[5] Detailed description of the changes and the calculation can be found on the CBOE website at http://www.cboe.com.

on the COBE website is from January 2, 1990. Our sample period ended on December 31, 2003. We use daily closing index levels and study both series in the paper.

To match the sample period of the VIX series, we create a sub-series whose sample period is from January 2, 1990 to December 31, 2003 for the VXO series. For each series we examine both the full sample and a sample that excludes observations greater than 3 standard deviations from the means. Observations greater than 3 standard deviations are reported in Table 1. These high implied volatility readings encompass brief periods of market stress such as the two-month post-crash period starting October 19, 1987, and the first three days after the market reopened following the September 11, 2001 terrorist attacks in the United States, as well as spikes in market volatility in 1988, 1998 and 2002.

<Table 1 is about here>

IV. Structural Change Test Results

Table 2 reports Bai and Perron (1998) statistics for tests of structural change in the mean value of the VXO and VIX series. For both series and all sample periods, both double maximum statistics (UDmax and WDmax) are significant at conventional significance levels, so we have strong evidence of structural change in the mean level of the volatility. In addition both the $F(2|1)$ and the $F(1|0)$ statistics are significant at the 5% level or higher, while the $F(3|2)$, $F(4|3)$ are insignificant. This suggests two structural breaks (three regimes) for the volatility series.[6]

<Table 2 is about here>

[6] These statistics are interpreted as follows. The significance of $F(1|0)$ indicates that the null of 0 breaks is rejected in favor of the alternative of 1 break. The significance of $F(2|1)$ indicates that the mull of 1 break is rejected in favor of the alternative of 2 breaks. However, the null of 2 breaks, $F(3|2)$, cannot be rejected.

Table 3 reports the dates for the structural breaks in the mean level of the volatility series and their 90% and 95% confidence intervals for each of the break dates. The break dates given correspond to the end of each regime. In addition, the average (mean) value of volatility is reported for each regime. For the VXO full sample period (1986-2003) we find two break dates; March 11, 1991 and July 16, 1997. When 3 standard deviations are excluded from VXO we find break dates at April 11, 1991 and June 12, 1997. For VXO, for the shorter January 2, 1990 to December 31, 2003 period, we find break dates at February 10, 1992 and February 18, 1997. When we exclude observations greater than 3 standard deviations from the mean we find break dates at February 10, 1992 and January 24, 1997. With respect to the VIX series for the January 2, 1990 to December 31, 2003 period, we find break dates at March 16, 1992 and July 17, 1997. When observations 3 standard deviations from the mean are excluded we find break dates at March 16, 1992 and July 16, 1997. The 90% and 95% confidence intervals for the break dates are reported below the break dates. These range between 1-2 years of the estimated break date.

<Table 3 is about here>

Based on the break dates, the volatilities series can be separated into different regimes. For the VXO full sample period including the 1987 market crash, regime one corresponds to the January 2, 1986 to March 11, 1991 period, regime two corresponds to the March 12, 1991 to July 16, 1997 period, and regime three corresponds to the July 17, 1997 to December 31, 2003 period. Excluding observations greater than 3 standard deviations from the mean, the three regimes are, January 2, 1986 to April 11, 1991, April 12, 1991 to June 12, 1997, and June 13, 1997 to December 31, 2003, respectively. The differences in the begin- and end- dates of the regimes are small. For the data series that excludes the 1987 market crash we begin the sample in January 2, 1990. The three regimes for the VIX series are January 2, 1990 to March 16, 1992, March 17, 1992 to July 17, 1997, and July 18, 1997 to December 31, 2003. The three regimes for the VXO series are January 2, 1990 to February 10, 1992, February 11, 1992 to February 18, 1997, and February 19, 1997 to December 31, 2003. Note that while the VIX and VXO are calculated with distinctly different methodologies, the begin- and end- dates of the regimes are rather consistent. Market volatility based on the VIX and VXO series can be roughly separated into three time periods; a pre-1991 period, a 1991-1997 period, and a 1997-2003 period when the 1987 market crash is included, and a 1990-1992 period, a 1992-1997 period and a post- 1997 period when our sample begins in 1990.

Table 4 shows the differences in the mean level and standard deviation of volatility series for these three regimes. For the VXO full sample period, the means (standard deviations) for the three regimes and the entire sample period are 23.13 (9.46), 15.34 (3.23), 26.67 (6.19) and 21.70 (8.19) respectively. Excluding observations greater than 3 standard deviations from the mean, the means (standard deviations) for the three regimes and the entire sample period are 22.22 (5.16), 15.18 (3.12), 26.43 (5.85) and 21.31 (6.81). For the VXO sub-series starting from 1990, the means (standard deviations) for the three regimes and the entire sample period are 20.68 (4.60), 14.39 (2.69), 26.40 (4.77) and 21.22 (7.35) while those for the VIX series are 20.45 (4.71), 14.63 (2.79), 24.76 (5.45) and 20.20 (6.45). The means and standard deviations for the two samples excluding observations greater than 3 standard deviations from the mean are essentially the same as those in their corresponding full samples. To summarize, these numbers consistently show that mean volatility fell substantially from regime one (pre-1991 or 1992) to regime two (1991 or 1992-1997), and then increased sharply during regime three (1997-2003) to a level that exceeded the mean volatility in regime one (the pre-1991 or 1992 period). It is interesting to note that both the means and standard deviations are lower during regime two

(1991 or 1992-1997) than during regime one (pre-1991 or 1992) and regime three (post 1997). The evidence suggests that, consistent with Schwert (1998), there is a period after the 1987 market crash the stock market volatility has been unusually low. The evidence further suggests that the means of market volatility proxied by the VIX and VXO series have not been stable during our sample periods. Figures I and II provide graphical depictions of the means of the three regimes identified for the VXO and VIX full sample series derived from our Bai and Perron (1998, 2003a) procedure.

<Table 4 and Figures I and II about here>

V. Summary

Market volatility indexes VIX and VXO are popular indicators of short-term market volatility. Traders use up-to-the-minute estimates of expected volatility to gauge investor fear (Whaley 2000). An ocular inspection of a graph of volatility indexes over time indicates that volatility has undergone infrequent, but significant shifts in its average level. In this paper, we use the Bai and Perron (1998, 2003a) method to test for multiple structural breaks in the mean levels of the volatility indexes based on the data provided by the Chicago Board Options Exchange. The available data series for VIX is from 1990-2003 and for VXO is from 1986-2003. We find evidence of three distinct periods for the data series we examined. For the VXO data series starting from 1986, the regimes correspond to a pre-1991 period, a 1992-1997 period, and a post-1997 period. When the data series start from 1990, the regimes correspond to a pre-1992 period, a 1992-1997 period, and a post-1997 period for both the VXO and VIX series. The regimes for the VXO and the VIX are quite similar despite the fact that the two indexes are calculated using different methodologies. The evidence suggests that the average volatility (and its standard deviation) are lowest during the 1992-1997 period, and the means of market volatility, proxied by the VIX and VXO, have not been stable during the sample periods.

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TABLE 1: Observations exceeding 3 standard deviations from the means in the VXO and VIX series

	VXO (1986-2003)		VXO (1990-2003)		VIX (1990-2003)
Mean (std.)	21.70 (8.19)		21.22 (7.35)		20.20 (6.45)
Mean +/-3 std.	(-2.88, 46.29)		(-0.81, 43.26)		(0.86, 39.54)
Date	VXO	Date	VXO	Date	VIX
10/10/02	46.29	10/02/02	43.36	08/28/98	39.60
10/01/98	46.45	10/12/98	43.41	09/09/98	39.66
10/08/02	46.46	10/11/02	43.44	09/23/02	39.68
11/16/87	46.50	07/19/02	43.45	09/30/02	39.69
11/18/87	46.53	09/03/02	43.86	07/24/02	39.86
12/15/87	47.01	10/06/98	43.91	09/03/02	39.97
12/03/87	47.22	09/03/98	43.93	10/12/98	40.07
12/14/87	47.44	09/11/98	44.09	10/13/98	40.23
09/10/98	47.52	10/05/98	44.44	10/02/98	40.47
11/13/87	47.59	09/20/02	44.55	09/24/02	40.52
11/20/87	47.62	09/30/02	44.57	09/19/01	40.56
12/04/87	47.63	09/04/98	44.61	09/19/02	40.65
07/22/02	48.23	07/25/02	44.65	09/30/98	40.95
10/07/98	48.27	09/23/02	44.71	10/08/02	41.02
09/21/01	48.27	09/17/01	44.94	10/06/98	41.20
08/31/98	48.33	10/03/02	44.96	08/02/02	41.29
12/11/87	48.44	07/24/02	45.29	09/03/98	41.43
10/08/98	48.56	09/24/02	45.38	09/17/01	41.76
12/10/87	49.00	08/02/02	45.39	10/09/98	41.78
09/20/01	49.04	10/09/98	45.69	07/22/02	41.87
10/07/02	49.18	08/06/02	45.73	08/06/02	42.03
11/12/87	49.21	09/19/02	46.16	10/09/02	42.13
08/05/02	49.31	10/04/02	46.28	10/07/02	42.64
01/08/88	49.36	10/10/02	46.29	09/21/01	42.66
10/09/02	49.48	10/01/98	46.45	10/05/98	42.81
11/19/87	49.50	10/08/02	46.46	09/04/98	43.31
11/17/87	49.59	09/10/98	47.52	10/01/98	43.48
07/23/02	50.48	07/22/02	48.23	10/07/98	43.51
11/05/87	53.54	10/07/98	48.27	09/11/98	43.74
11/09/87	54.73	09/21/01	48.27	09/20/01	43.74
11/10/87	54.76	08/31/98	48.33	08/31/98	44.28
11/02/87	54.90	10/08/98	48.56	07/23/02	44.92
11/11/87	55.55	09/20/01	49.04	08/05/02	45.08
11/06/87	55.78	10/07/02	49.18	09/10/98	45.29
11/03/87	58.44	08/05/02	49.31	10/08/98	45.74
11/04/87	58.47	10/09/02	49.48		
10/30/87	61.41	07/23/02	50.48		
10/29/87	64.66				
10/21/87	73.91				
10/28/87	81.24				
10/27/87	97.51				
10/23/87	98.81				
10/22/87	102.22				
10/26/87	113.33				
10/20/87	140.00				
10/19/87	150.19				

Notes: This table reports volatility indexes VIX and VXO at levels greater than 3 standard deviations from the mean. The VIX is the Chicago Board Options Exchange (CBOE) volatility index based on options on the S & P 500 index and the VXO is based on options on the S & P 100 index. We extracted daily closing index levels up to December 31, 2003 from the CBOE website at www.cboe.com. Data available for the VXO series is from January 2, 1986 and for VIX is from January 2, 1990. To match the sample period of the VIX series, we create a sub-series whose sample period is from January 2, 1990 to December 31, 2003 for the VXO series.

TABLE 2: Bai and Perron statistics for tests of multiple structural breaks for the VXO and VIX Series

Series	UDmax[a]	WDmax(5%)[b]	F(1|0)[c]	F(2|1)[d]	F(3|2)[e]	F(4|3)[f]	F(5|4)[g]
VXO1 01/02/86-12/31/03	55.08***	71.48**	24.22***	23.41***	1.62	1.68	0.06
VXO1-3std. dev. 01/02/86-12/31/03	60.21***	75.98**	24.01***	27.05***	1.70	0.44	0.07
VXO2 01/02/90-12/31/03	57.19***	90.56**	42.05***	16.11***	2.57	0.38	-
VXO2-3std. dev. 01/02/90-12/31/03	65.71***	112.93**	44.01***	16.87***	2.57	0.41	-
VIX 01/02/90-12/31/03	56.84***	71.90**	34.59***	13.47**	1.15	0.08	0.04
VIX-3std. dev. 01/02/90-12/31/03	63.82***	88.30**	35.24***	13.52**	1.15	0.10	0.02

Notes: Data are from the Chicago Board Options Exchange (CBOE).
VXO is the CBOE volatility index based on options on the S&P 100 Index (OEX).
VXO1 indicates the use of the VIX for the January 2, 1986 to December 31, 2003 period.
VXO1-3std. dev. indicates the use of the VXO1 sample period but excludes observations above 3 standard deviations from the mean.
VXO2 indicates the use of the VIX for the sample January 2, 1990 to December 31, 2003 period.
VXO2-3std. dev. indicates the use of the VXO2 sample but excludes observations above 3 standard deviations from the mean.
VIX is the CBOE volatility index based on options on the S&P 500 index (SPX) and covers the January 2, 1990 to December 31, 2003 period.
VIX-3std. dev. indicates the use of the VIX sample period but excludes observations above 3 standard deviations from the mean.
[a]One-sided (upper-tail) test of the null hypothesis of 0 breaks against the alternative hypothesis of an unknown number of breaks given an upper bound of 5; 10, 5, and 1 percent critical values equal 7.46, 8.88, and 12.37, respectively.
[b]One-sided (upper-tail) test of the null hypothesis of 0 breaks against the alternative hypothesis of an unknown number of breaks given an upper bound of 5; critical value equals 9.91.
[c]One-sided (upper-tail) test of the null hypothesis of 0 breaks against the alternative hypothesis of 1 break; 10, 5, and 1 percent critical values equal 7.04, 8.58, and 12.29, respectively.
[d]One-sided (upper-tail) test of the null hypothesis of 1 break against the alternative hypothesis of 2 breaks; 10, 5, and 1 percent critical values equal 8.51, 10.13, and 13.89, respectively.
[e]One-sided (upper-tail) test of the null hypothesis of 2 breaks against the alternative hypothesis of 3 breaks; 10, 5, and 1 percent critical values equal 9.41, 11.14, and 14.80, respectively.
[f]One-sided (upper-tail) test of the null hypothesis of 3 breaks against the alternative hypothesis of 4 breaks; 10, 5, and 1 percent critical values equal 10.04, 11.83, and 15.28, respectively.
[g]One-sided (upper-tail) test of the null hypothesis of 4breaks against the alternative hypothesis of 5 breaks; 10, 5, and 1 percent critical values equal 10.58, 12.25, and 15.76, respectively.
*** Significant at the 1% level
** Significant at the 5% level
* Significant at the 10% level
- indicates that there was no more place to insert an additional break given the minimal length requirement.

TABLE 3: Bai and Perron regime means, end dates, and 90% and 95% confidence intervals for the VXO and VIX series

Series	Regime 1	Regime 2	Regime 3
VXO1	23.13 (1.46)	15.34 (0.76)	26.67 (1.12)
01/02/86-12/31/03			
End Date	3/11/91	7/16/97	
90% CI	[1/7/90, 8/26.92]	[1/7/97, 9/22/97]	
95% CI	[10/17/90, 5/12/93	[12/4/96, 12/10/97]	
VXO1-3std. dev.	22.22 (1.15)	15.18 (0.72)	26.43 (1.05)
01/02/86-12/31/03			
End Date	4/11/91	6/12/97	
90% CI	[11/7/90, 6/1/92]	[12/23/96,8/8/97]	
95% CI	[7/10/90, 9/29/92]	[8/28/96, 7/24/97]	
VXO2	20.68 (1.46)	14.39 (0.58)	26.40 (1.09)
01/02/90-12/31/03			
End Date	2/10/92	2/18/97	
90% CI	[11/7/91, 1/21/93]	[8/23/96, 3/7/97]	
95% CI	[9/19/91, 6/8/93]	[6/17/96, 3/20/97]	
VXO2-3std. dev.	20.68 (1.46)	14.30 (0.54)	25.93 (0.93)
01/02/90-12/31/03			
End Date	2/10/92	1/24/97	
90% CI	[11/29/91, 1/13/93]	[9/9/96, 1/17/97]	
95% CI	[1/2/92, 3/1/96]	[10/26/99, 7/13/00]	
VIX	20.45 (1.45)	14.63 (0.64)	24.76 (0.96)
01/02/90-12/31/03			
End Date	3/16/92	7/17/97	
90% CI	[10/14/91, 5/4/93]	[1/16/97, 9/2/97]	
95% CI	[7/29/91, 10/18/93]	[11/5/96, 9/25/97]	
VIX-3std. dev.	20.45 (1.45)	14.62 (0.64)	24.37 (0.85)
01/02/90-12/31/03			
End Date	3/16/92	7/16/97	
90% CI	[10/16/91, 5/4/93]	[2/20/97, 9/8/97]	
95% CI	[8/1/91, 10/18/93]	[12/20/96, 10/3/97]	

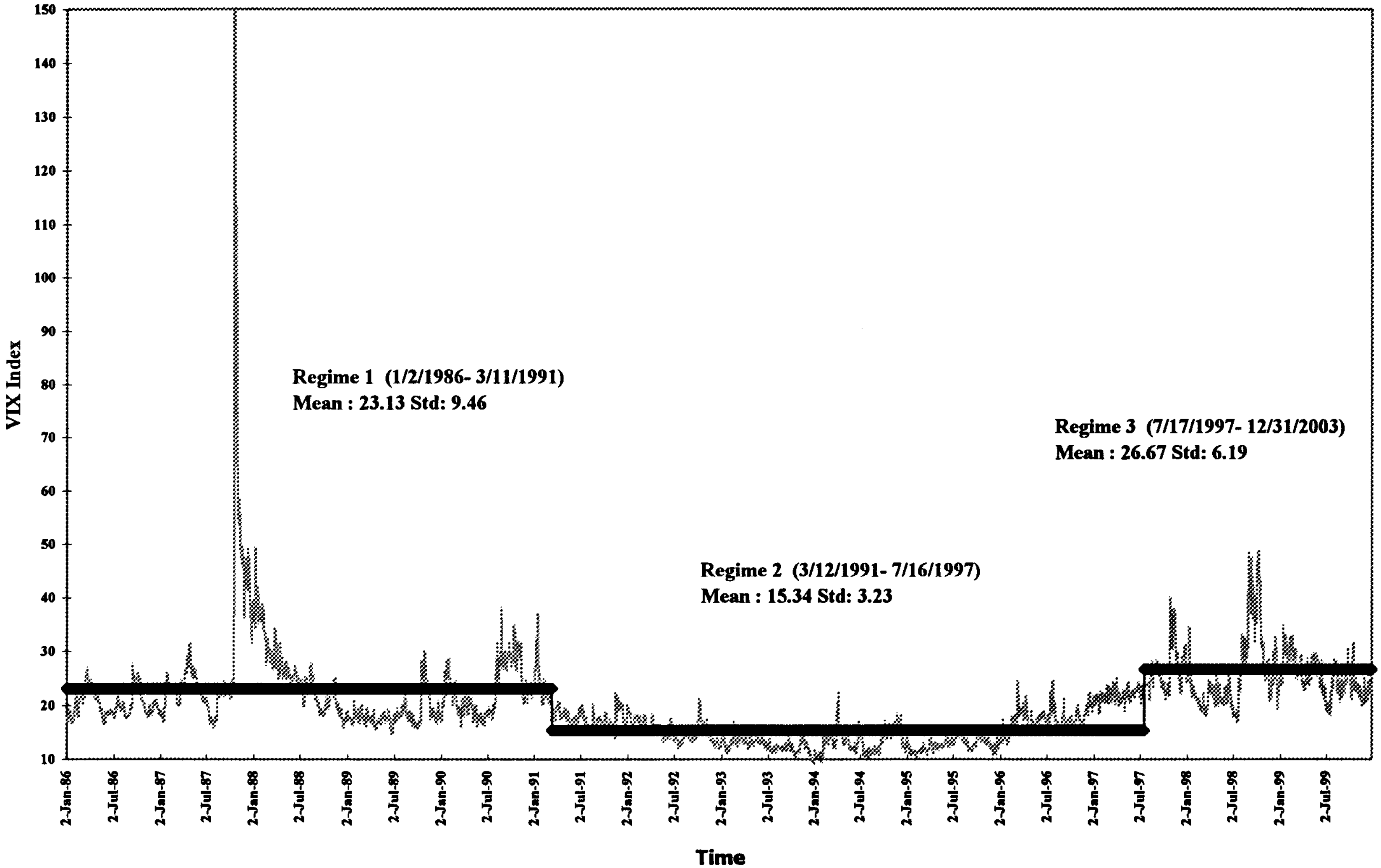
Notes: Data are from the Chicago Board Options Exchange (CBOE).
VXO is the CBOE volatility index based on options on the S&P 100 Index (OEX).
VXO1 indicates the use of the VIX for the January 2, 1986 to December 31, 2003 period.
VXO1-3std. dev. indicates the use of the VXO1 sample period but excludes observations above 3 standard deviations from the mean.
VXO2 indicates the use of the VIX for the sample January 2, 1990 to December 31, 2003 period.
VXO2-3std. dev. indicates the use of the VXO2 sample but excludes observations above 3 standard deviations from the mean.
VIX is the CBOE volatility index based on options on the S&P 500 index (SPX) and covers the January 2, 1990 to December 31, 2003 period.
VIX-3std. dev. indicates the use of the VIX sample period but excludes observations above 3 standard deviations from the mean.
The first number in each cell is the estimated mean for the regime; standard errors are reported in parentheses. The end date for the regime is below the mean; 90% and 95% confidence intervals for the end dates are reported in brackets.

TABLE 4: Means and standard deviations in the regimes for the VXO and VIX series

	Regime 1	Regime 2	Regime 3	Entire period
Panel A	**VXO series**			
VXO1 (01/2/86-12/31/03)				
Time periods	1/2/86-3/11/91	3/12/91-7/16/97	7/17/97-12/31/03	1/2/86-12/31/03
Mean	23.13	15.34	26.67	21.70
Std. Dev.	9.46	3.23	6.19	8.19
Maximum	150.19	24.99	50.48	150.19
Minimum	14.76	9.04	15.35	9.04
VXO1-3std. dev. (01/02/86-12/31/03)				
Time periods	1/2/86-4/11/91	4/12/91-6/12/97	6/13/97-12/31/03	1/2/86-12/31/03
Mean	22.22	15.18	26.43	21.31
Std. Dev.	5.16	3.12	5.85	6.81
Maximum	46.13	24.99	46.28	46.28
Minimum	14.76	9.04	15.35	9.04
VXO2 (01/02/90-12/31/03)				
Time periods	1/2/90-2/10/92	2/11/92-2/18/97	2/19/97-12/31/03	1/2/90-12/31/03
Mean	20.68	14.39	26.40	21.22
Std. Dev.	4.60	2.69	4.77	7.35
Maximum	38.07	24.43	50.48	50.48
Minimum	13.93	9.04	15.35	9.04
VXO2-3std. dev. (01/02/1990-12/31/03)				
Time periods	1/2/90-2/10/92	2/11/92-1/24/97	1/25/97-12/31/03	1/2/90-12/31/03
Mean	20.68	14.30	25.93	20.96
Std. Dev.	4.60	2.59	5.41	6.92
Maximum	38.07	24.43	43.24	43.24
Minimum	13.93	9.04	15.35	9.04
Panel B	**VIX series**			
VIX (01/02/90-12/31/03)				
Time periods	1/2/90-3/16/92	3/17/92-7/17/97	7/18/97-12/31/03	1/2/90-12/31/03
Mean	20.45	14.63	24.76	20.20
Std. Dev.	4.71	2.79	5.45	6.45
Maximum	36.47	23.87	45.74	45.74
Minimum	13.95	9.31	15.58	9.31
VIX-3std. dev. (01/02/1990-12/31/03)				
Time periods	1/2/90-3/16/92	3/17/92-7/16/97	7/17/97-12/31/03	1/2/90-12/31/03
Mean	20.45	14.62	24.37	19.99
Std. Dev.	4.71	2.79	4.86	6.10
Maximum	36.47	23.87	39.46	39.46
Minimum	13.95	9.31	15.58	9.31

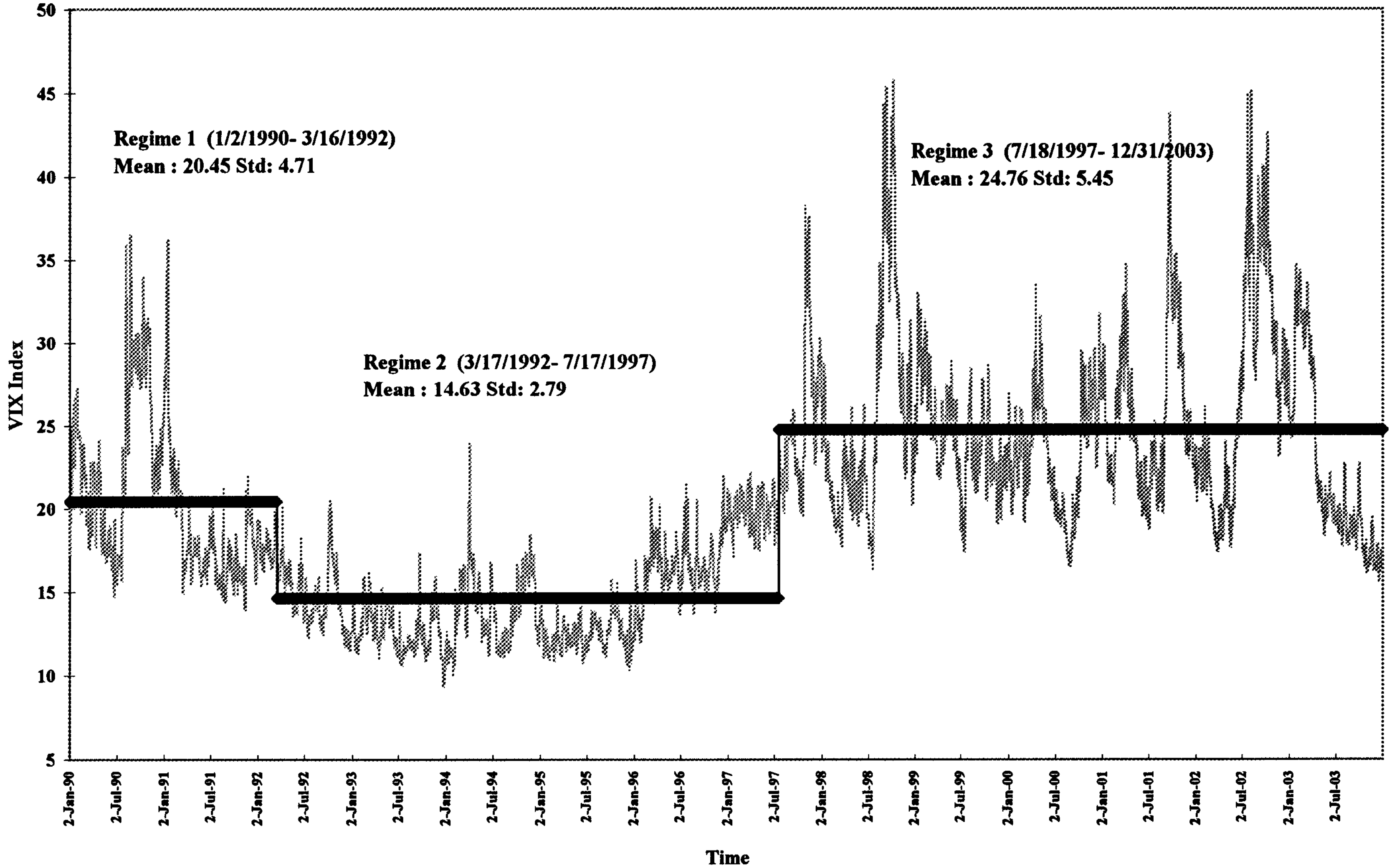
Notes: Data are from the Chicago Board Options Exchange (CBOE).
VXO is the CBOE volatility index based on options on the S&P 100 Index (OEX).
VXO1 indicates the use of the VIX for the January 2, 1986 to December 31, 2003 period.
VXO1-3std. dev. indicates the use of the VXO1 sample period but excludes observations above 3 standard deviations from the mean.
VXO2 indicates the use of the VIX for the sample January 2, 1990 to December 31, 2003 period.
VXO2-3std. dev. indicates the use of the VXO2 sample but excludes observations above 3 standard deviations from the mean.
VIX is the CBOE volatility index based on options on the S&P 500 index (SPX) and covers the January 2, 1990 to December 31, 2003 period.
VIX-3std. dev. indicates the use of the VIX sample period but excludes observations above 3 standard deviations from the mean.

Figure 1. VXO Index (OEX-based) and Structural Breaks (1986-2003)



This figure reports the structural breaks in the mean level of market volatility reflected in the Chicago Board Options Exchange (CBOE) volatility index VXO (based on options on the S & P 100 index) for the January 2, 1986 to December 31, 2003 period. We find evidence of three distinct periods. The periods correspond to a pre-1991 period, a 1991-1997 period, and a post-1997 period. The dates of these mean shifts as well as the mean and standard deviation of the volatility levels are reported in the figure.

Figure 2. VIX Index (SPX-based) and Structural Breaks (1990-2003)



This figure reports the structural breaks in the mean level of market volatility reflected in the Chicago Board Options Exchange (CBOE) volatility index VIX (based on S & P 500 index options) for the January 2, 1990 to December 31, 2003 period. We find evidence of three distinct periods. The periods correspond to a pre-1992 period, a 1992-1997 period, and a post-1997 period. The dates of these mean shifts, as well as the mean and standard deviation of the volatility levels are reported in the figure.