

Market Fundamentals vs Rational Bubbles in Stock Prices: A Bayesian Perspective

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Abstract

In this paper, we decompose the log price dividend ratio into a market fundamentals component and a bubble component. The market fundamentals component depends on expectations of future dividend growth and required returns while the bubble component is assumed to follow a Markov switching model that allows for the possibility of exploding and collapsing regimes. We take an explicitly Bayesian approach in specifying the behavior of the market fundamentals and bubble component. This allows us to investigate within a common framework how sensitive inference about the bubble is to prior beliefs concerning the nature of the market fundamentals and the bubble. We estimate the posterior distribution of the relative contributions of market fundamentals and rational bubble to movements in the log price dividend ratio using Markov Chain Monte Carlo techniques and find that one's prior beliefs have a substantial effect on the posterior distribution. In particular, if prior beliefs allow for the possibility of persistent (albeit small) shocks to dividend growth and/or required returns, the posterior distribution suggests the bubble component contributes virtually nothing to the stock price movements over our sample. On the other hand, if one's priors rule out the possibility of persistent shocks to dividend growth and required returns, the bubble component can have a much larger role to play in stock price movements. However, the regime switching behavior of the bubble bears little resemblance to infrequent switching from an exploding bubble regime to a collapsing or dormant bubble regime.

Introduction

Did the dramatic rise in stock prices during the 1990s reflect a bubble or were stock prices responding to new information about future cash flows and discount rates? While there is a large literature suggesting stock prices are too volatile to be driven solely by market fundamentals (see for example work of Shiller 1981, 2003; Blanchard and Watson, 1982 and many others), LeRoy (2004, p. 785) suggests in his recent literature review that the empirical evidence for bubbles is inconclusive as he notes that the existence of bubbles is ultimately "...an empirical question, and the empirical literature on bubbles is not yet well developed." The identification of a stock price bubble is made difficult because two of the primary fundamental factors that contribute to movements in stock prices (required returns and expected future dividend growth) are unobserved stochastic processes.¹

Unlike other types of departures from market fundamentals, a rational bubble imposes fairly strong restrictions on the time series properties of the asset price which can in principle be used to identify a bubble in the data—no amount of differencing will induce the asset price to become stationary (Hamilton and Whiteman (1985) and Diba and Grossman (1988)). While existing tests in the literature have attempted to exploit these restrictions by testing the (non) stationarity of asset prices, these tests in practice are plagued by low power and size distortions in small samples. For example, a bubble that is intermittently active then bursts and becomes dormant can be difficult to detect with traditional tests of nonstationarity.²

¹ The difficulty posed by unobserved market fundamentals for the detection of rational bubbles has been pointed out by Flood and Garber (1980), Hamilton and Whiteman (1985), and many others.

² Evans (1991), Charemza and Deadman (1995) and more recently Wu and Xiao (2002) report results that indicate that unit root tests are unable to detect collapsing bubbles. For a selection of empirical studies that employ both unit root tests, cointegration tests, and fractional integration tests on stock prices and dividends over various sample periods see, Froot and Obstfeld, 1991; Craine, 1993; Crowder and Wohar (1998), Lamont, 1998; Caporale and Gil-Alana, 2004; Koustas and Serletis, 2005 and Cunado, Gil-Alana, and De Gracia, 2005).

In this paper, rather than test whether a bubble exists or not, we attempt to decompose asset prices into a market fundamental component and a bubble component. That is, we attempt to identify and extract the bubble directly from the data. This requires on our part taking a stand on the form of the bubble and the nature of the market fundamentals. We take a Bayesian approach to decomposing stock prices into market fundamentals and a bubble component. The objective is to estimate the posterior distribution of the relative contributions of market fundamentals and rational bubble to movements in the log price dividend ratio. The Bayesian approach allows us to investigate in a common framework how sensitive inferences about the bubble are to prior beliefs about the nature of the market fundamentals and the dynamic process describing the bubble.

Specifically, we consider a model where dividend growth and required returns include both transitory and persistent components. We also allow the bubble component to temporarily expand or burst and collapse by specifying a Markov regime switching model for the bubble component. As none of the underlying determinants of the price dividend ratio (required return, expected dividend growth, or the bubble) are observed directly we use a state-space/unobserved components modeling approach to estimate the relative contribution of the market fundamental components and a bubble factor to movements in stock prices.

Our model is general enough to nest within it the specifications of market fundamentals and bubbles made in previous studies. For example, Shiller (1981), in his seminal paper on excess volatility in stock prices, considers a simple present-value model of stock prices in which only a stationary dividend growth factor affects stock prices. This model is a special case of very strong prior beliefs about the nature of the market fundamentals—no persistent factor in dividend

growth and constant required returns.³ Subsequent studies have considered a wide range of alternative assumptions about the behavior of dividends and returns and we view these alternative assumptions in our analysis as having strong priors about the nature of the market fundamentals and the bubble.⁴

We estimate and summarize the posterior distribution of the relative contributions of market fundamentals and bubble component for various cases of prior beliefs about the nature of the market fundamentals and bubble volatility. Our results indicate that once one allows for a persistent factor in dividend growth and/or required returns, the posterior distribution of the bubble's contribution to stock price movements is negligible. Only when prior beliefs rule out the presence of persistent changes (even relatively small changes) in market fundamentals, does the bubble component play a significant role in explaining stock price movements. Even in these cases where the bubble is an important contributor to stock price movements, the implied behavior of the bubble does not exhibit the type of expanding/collapsing behavior that many analysts might expect a priori. For example, there is little evidence that the run up in stock prices during the 1990s and the subsequent decline was due to being in an expanding bubble regime which burst in the early 2000s.

³ Shiller, in this paper, also considers the case where variation in the required return was captured by a short term interest rate, implicitly assuming that excess returns rather than required returns are constant.

⁴ Campbell and Shiller (1988) and Campbell and Ammer (1993) decompose log price dividend into expectations of future dividends and returns and assume the market fundamentals are stationary and no bubble exists. Balke and Wohar (2002, 2006) allow for permanent factors in dividend growth or returns but also do not allow for a bubble. Wu (1997) decomposes stock prices into market fundamentals and bubble component but assumes constant required returns and temporary dividend growth. van Norden and Schaller (1999) and Brooks and Katsaris (2005) assume stationary market fundamentals and model the bubble switching between expanding, collapsing, and dormant regimes. See Gurkaynak (2005) for a review of econometric tests of asset pricing bubbles.

2. Log price dividend ratio, market fundamentals, and rational bubbles

Most of previous literature has examined a bubble in the context of the level of stock prices not the log price dividend ratio (see Wu (1997) as an exception). However, most of these models assumed constant required returns for stocks in order retain linear structure of the model. Here we consider a bubble in the Campbell and Shiller (1988) approximation. This enables us to allow for time varying required returns and yet retain a linear structure which vastly simplifies the econometric analysis below.

The Campbell and Shiller (1988) approximation is:

$$(1) \quad p_t = \rho p_{t+1} + d_{t+1} - r_{t+1} + k, \quad 0 < \rho < 1$$

where p_t is the log(price/dividend), d_t is dividend growth, r_t = rate of return on stock, and k is a constant.⁵ The parameter $\rho = \exp(\bar{p}) / (1 + \exp(\bar{p}))$ where \bar{p} is the sample mean of the log price dividend ratio.

Taking expectations, yields:

$$(2) \quad p_t = \rho E_t p_{t+1} + E_t d_{t+1} - E_t r_{t+1} + k$$

Solving (2) forwards results in

$$(3) \quad p_t = \frac{k}{1-\rho} + \sum_{i=0}^{\infty} \rho^i E_t [d_{t+1+i} - r_{t+1+i}] + \lim_{k \rightarrow \infty} \rho^k E_t p_{t+k}$$

We define the market fundamentals as

$$(4) \quad p_t^{\text{mf}} = \frac{k}{1-\rho} + \sum_{i=0}^{\infty} \rho^i E_t [d_{t+1+i} - r_{t+1+i}].$$

If a bubble is present then we can write the log(price/dividend) as:

⁵ Both dividend growth and required rate of return on stocks are in nominal values. We leave them in nominal values rather than the more common used real values as the term $d_{t+1} - r_{t+1}$, which is the key market fundamental factor, is the same regardless of whether we use nominal dividends and nominal returns or use real dividend and real returns.

$$(5) \quad p_t = p_t^{\text{mf}} + b_t,$$

where b_t denotes a bubble. The bubble, in turn, must satisfy following equation:

$$(6) \quad E_t b_{t+1} = \left(\frac{1}{\rho}\right) b_t.$$

The condition $\lim_{k \rightarrow \infty} \rho^k E_t p_{t+k} = 0$ in equation (3) is sufficient to rule out the presence of a bubble.

2.1 Regime switching model for the bubble

In specifying the dynamics of the bubble, the key constraint for the dynamics is that it must satisfy equation (6).⁶ We do not, however, want to restrict the bubble to follow a linear process. In particular, we want our model to be able to capture bubbles that can burst periodically and then reappear. As a result, we allow for two separate bubble regimes: an expanding or exploding regime and a non-exploding or collapsing regime.⁷ We denote the variable, I_{t+1} , to indicate which type of regime the bubble is in; $I_{t+1} = 1$ denotes the exploding bubble regime and $I_{t+1} = 0$ denotes the non-exploding bubble regime. A Markov switching model governs the evolution of the bubble regime. The probability of continuing in the exploding regime is given by $p[I_{t+1} = 1 | I_t = 1] = q$, while the probability of continuing in the non-explosive regime is given by $p[I_{t+1} = 0 | I_t = 0] = s$.

When the bubble is in the non-exploding regime, $I_{t+1} = 0$, we assume it follows the following equation:

⁶ An additional restriction on the bubble discussed in the literature is that it must be positive. That restriction takes on less force here as we are examining $\log(p/d)$ rather the level of stock prices. We do consider that restriction when conducting sensitivity analysis of our results.

⁷ Among the many papers that examine periodically collapsing bubbles are Blanchard (1979), Blanchard and Watson (1982), Evans (1991), van Norden and Schaller (1993), van Norden (1996), and Brooks and Katsaris (2005).

$$(7) \quad b_{t+1} = \gamma b_t + \bar{b} + v_{t+1}^b,$$

where $0 < \gamma < 1$. This specification allows for a slowly collapsing bubble. When the bubble is continuing in its expanding regime ($I_{t+1} = 1, I_t = 1$),

$$(8) \quad b_{t+1} = \frac{1}{q} \left[\frac{1}{\rho} - (1-q)\gamma \right] b_t - \frac{(1-q)\bar{b}}{q} + v_{t+1}^b$$

Finally, when in the period in which the bubble switches from a non-explosive regime to an explosive regime ($I_{t+1} = 1, I_t = 0$), it is given by the equation:

$$(9) \quad b_{t+1} = \frac{1}{1-s} \left[\frac{1}{\rho} - r\gamma \right] b_t - \frac{s}{1-s} \bar{b} + v_{t+1}^b.$$

Note that the conditional expectation of the bubble at $t+1$ given t information is $E_t b_{t+1} = \frac{1}{\rho} b_t$.⁸

2.2 Market fundamentals

For the market fundamentals, we specify unobserved components models for dividend growth and the returns processes similar to that found in Balke and Wohar (2002). For dividend growth:

$$(10) \quad d_t = d_t^p + d_t^\tau + w_t^d,$$

where d_t^p is a persistent dividend growth factor, d_t^τ is a transitory or temporary dividend growth factor, and w_t^d reflects a component of measured dividend growth that does not affect stock prices (i.e. measurement error). For parsimony and clarity, we model the persistent component of dividend growth as a random walk

$$(11) \quad d_t^p = d_{t-1}^p + v_t^{d,p},$$

⁸ This assumes that market participants know the regime at time t , but not at $t+1$.

although we also consider below a model where the persistent component is a stationary autoregression. The temporary dividend growth factor is given by

$$(12) \quad d_t^\tau = \theta_d(L)d_{t-1}^\tau + v_t^{d,\tau},$$

where $\theta_d(L) = \sum_{i=1}^m \theta_{d,i} L^{i-1}$ is a lag polynomial whose roots are outside the unit circle. Similarly,

there is a persistent factor for required returns,

$$(13) \quad r_t^p = r_{t-1}^p + v_t^{r,p},$$

and a transitory or temporary factor for required returns,

$$(14) \quad r_t^\tau = \theta_r(L)r_{t-1}^\tau + v_t^{r,\tau}$$

with $\theta_r(L) = \sum_{i=1}^m \theta_{r,i} L^{i-1}$.

We allow for the possibility that innovations in the market fundamentals components ($v_t^{d,p}$, $v_t^{d,\tau}$, $v_t^{r,p}$, $v_t^{r,\tau}$) are correlated. In fact, we would expect that innovations in these components to be correlated. For example, an increase in expectations of future dividend growth that are not fully reflected in current dividend growth imply a negative relationship between $v_t^{d,p}$ and $v_t^{d,\tau}$. Also a consumption based asset pricing model suggests that a persistent change in dividend growth is likely to be accompanied by a persistent change in the required returns.⁹

⁹ A persistent increase in dividend growth that in turn results in a persistent increase in consumption growth lowers the stochastic discount factor or equivalently raises the required return.

2.3 State space model for stock prices

The market fundamentals components ($d_t^p, d_t^\tau, r_t^p, r_t^\tau$) and the bubble (b_t) correspond to the underlying state variables in the state space model while the log price dividend ratio and dividend growth reflect the observable variables. As none of the underlying determinants of the log price dividend ratio are observed directly, we adopt a state space/unobserved components approach to estimating the relative contribution of market fundamentals and bubble to stock price movements. While we do observe dividend growth, however, that observation contains some measurement error. Similarly, we assume that the log price dividend ratio also contains measurement error.

Combining the bubble component with the market fundamentals components yields the following state equation:

$$(15) \quad S_t = \delta_t + F_t S_{t-1} + V_t$$

$$S_t = (b_t, d_t^p, r_t^p, d_t^\tau, r_t^\tau, d_{t-1}^\tau, r_{t-1}^\tau, \dots, d_{t-m+1}^\tau, r_{t-m+1}^\tau)'$$

$$\delta_t = \begin{pmatrix} \bar{b}_t \\ 0_{2(m+1) \times 1} \end{pmatrix} \text{ where } \bar{b}_t = \bar{b}(1 - I_t) - \frac{sr}{1-r} \bar{b} I_t (1 - I_{t-1}) - \frac{(1-q)\bar{b}}{q} I_t I_{t-1}$$

$$F_t = \begin{pmatrix} F_{b,t} & 0_{1 \times 2(m+1)} \\ 0_{2(m+1) \times 1} & F \end{pmatrix}$$

$$F_{b,t} = \gamma(1 - I_t) + \frac{1}{1-s} \left[\frac{1}{\rho} - s\gamma \right] I_t (1 - I_{t-1}) + \frac{1}{q} \left[\frac{1}{\rho} - (1-q)\gamma \right] I_t I_{t-1}$$

$$F = \begin{pmatrix} I_{2 \times 2} & 0_{2 \times 2m} \\ 0_{2m \times 2} & \begin{matrix} \Theta_1 & \dots & \Theta_m \end{matrix} \\ & \begin{matrix} I_{2(m-1) \times 2(m-1)} & 0_{2(m-1) \times 2} \end{matrix} \end{pmatrix}, \quad \Theta_i = \begin{pmatrix} \theta_{d,i} & 0 \\ 0 & \theta_{r,i} \end{pmatrix}$$

$$V_t \sim N(0, Q), \quad Q = \left(\begin{array}{c|cc} \sigma_b^2 & & \\ \hline & \mathbf{Q}_f & \mathbf{0}_{4 \times 2(m-1)} \\ \hline \mathbf{0}_{2(m+1) \times 1} & \mathbf{0}_{2(m-1) \times 4} & \mathbf{0}_{2(m-1) \times 2(m-1)} \end{array} \right)$$

$$\text{where } \mathbf{Q}_f = \mathbb{E} \left[\begin{array}{c} \left(\begin{array}{c} v_t^{d,p} \\ v_t^{r,p} \\ v_t^{d,\tau} \\ v_t^{r,\tau} \end{array} \right) \left(\begin{array}{cccc} v_t^{d,p} & v_t^{r,p} & v_t^{d,\tau} & v_t^{r,\tau} \end{array} \right) \end{array} \right].$$

Note that the intercept term and the autoregressive parameter for the bubble are time varying, reflecting the effect of regime switching for the bubble process.

Using equation (15) to describe the dynamics of the market fundamentals and the bubble, taking expectations and substituting into (4) and (5) yields the following observation equation:

$$(16) \quad \begin{pmatrix} p_t \\ d_t \end{pmatrix} = A + H S_t + W_t$$

$$\text{with } A = \begin{pmatrix} A_p \\ 0 \end{pmatrix}, \quad H = \begin{pmatrix} H_p \\ H_d \end{pmatrix}, \quad H_p = \left(\mathbf{1} \quad H_{pm} \left(\mathbf{I}_{2(m+1) \times 2(m+1)} - \rho F \right)^{-1} F \right),$$

$$H_{pm} = \left(\mathbf{1} \quad -\mathbf{1} \quad \mathbf{1} \quad -\mathbf{1} \quad \mathbf{0}_{1 \times 2(m-1)} \right), \quad H_d = \left(\mathbf{0} \quad \mathbf{1} \quad \mathbf{0} \quad \mathbf{1} \quad \mathbf{0} \quad \mathbf{0}_{1 \times 2(m-1)} \right), \quad \text{and } W_t = \begin{pmatrix} w_t^p \\ w_t^d \end{pmatrix}.$$

We denote the variance\covariance matrix of the noise vector, W_t , by R . We included an intercept term in the log price dividend equation to capture the effect of the constant k in the approximation and the contribution of the mean of the dividend growth process as we use demeaned real dividend growth data in the estimation. The market fundamentals component of stock prices is embedded in equation (16) and is given by:

$$(17) \quad p_t^{mf} = \sum_{i=0}^{\infty} \rho^i \mathbb{E}_t(d_{t+1+i} - r_{t+1+i}) = H_{pm} \left(\mathbf{I}_{2(m+1) \times 2(m+1)} - \rho F \right)^{-1} F S_t^{mf}$$

$$\text{with } S_t^{mf} = (d_t^p, r_t^p, d_t^\tau, r_t^\tau, d_{t-1}^\tau, r_{t-1}^\tau, \dots, d_{t-m+1}^\tau, r_{t-m+1}^\tau)' .$$

Equation (17) reflects the (rational) expectations of future dividend growth and realized returns as implied by the state space model. Note that unlike dividends, we have no direct observation on required returns or the bubble and must infer them solely from the log price dividend ratio.

One additional fact to note about equations (16) and (17) is the factor loadings for the permanent dividend growth and required returns components are $1/(1-\rho)$ and $-1/(1-\rho)$ respectively. Given the sample log price dividend ratio, the value of ρ is approximately 0.992 which implies factor loadings for the permanent components of the market fundamentals of around 125. This implies that a permanent change dividend growth or required returns will have a very large affect on the log price dividend ratio.

3. Bayesian Estimation of State Space Model

We estimate the model using Bayesian Markov Chain Monte Carlo (MCMC) methods. Essentially, we estimate the posterior distribution of the parameters and unobserved states by sequentially drawing a realization of the parameters from a conditional distribution of the parameters given the states and then drawing a realization of the states from the conditional distribution of the states given the parameters. The sample distribution of these draws will converge to the joint posterior distribution of the parameters and the states (see appendix for details).

In setting the prior distributions of the parameters, we generally try to choose relatively uninformative but proper priors; the details about the choice of prior distributions are in the appendix.¹⁰ The priors for the transition probabilities in the bubble regime imply fairly persistent

¹⁰ One way to interpret our choice of priors for some of the parameters such as the variances of shocks or the transition probabilities is that for “uninformed” priors we gave the prior distribution weight equal to five sample observations, the actual data being 214 observations, when computing the posterior distribution. For other parameters, we set prior distribution to have a “large” variance.

regimes (prior distribution of q and s were centered on 0.95) and an unconditional probability of being in the explosive regime of 0.5. We force the AR(2) models for temporary components of dividend growth and required returns to be stationary by accepting only parameter draws that imply stationary roots for the autoregressive model. Similarly, we restrict the dynamics in the non-explosive bubble regime by restricting draws for γ to be between zero and one.

Where we depart from uninformed priors is in setting prior distributions for the variances of shocks to the market fundamentals and the bubble. These variances are crucial to determining the relative size of the various determinants of stock prices. By choosing “tight” priors for specific variances, we can mimic the various assumptions about market fundamentals and bubbles that are present in the literature. In particular, by setting “tight” priors for specific variances we can replicate the different specifications that have been employed in the literature.

We consider the following cases in which we gradually relax very strong priors about the properties of the market fundamentals, namely that one or more components of the market fundamentals are absent. By dropping one or more of the market fundamental components, we essentially assign zero prior probability to a non-zero variance for that component.¹¹

Case 1. Required returns are constant and there is only a temporary component in dividend growth. Shiller (1981) in his original examination of excess volatility essentially makes these assumptions about the market fundamentals. In the context of the state space model, this corresponds to very strong prior beliefs that the variances of permanent components of dividend growth and required returns and the variance of the temporary component of required returns are zero.

¹¹ Computationally, we drop the component in question from the state space model.

Case 2. Required returns are time varying but only temporary components are present for both returns and dividend growth. This corresponds to the specification of market fundamentals employed by Campbell and Shiller (1987), Campbell and Ammer (1993), and Cochrane (1992).

Case 3. There is only one permanent factor in the market fundamentals. This allows a non-zero variance for one, but not both, of the permanent components of the market fundamentals. There are two sub-cases:

- a. The permanent component for dividend growth is present, but not for required returns, while both transitory components are present.
- b. The permanent component for required returns is present, but not for dividend growth, while both transitory components are present. Permanent changes in the equity premium would be reflected in permanent changes in the required returns.¹²

Case 4. Both permanent and transitory components for dividend growth and required returns are present. This relaxes all the strong priors on the market fundamentals variances, allowing both dividend growth and required returns to have permanent components and amounts to assuming “uniformed” priors for all the variances of shocks to the model.¹³

Table 1 presents the relative contribution to the variability of the log price dividend ratio of the market fundamentals components and the bubble component implied by the various prior distributions.¹⁴ To help understand the implications of Table 1, it will be instructive to discuss

¹² Barsky and DeLong (1993) examine case 3a while Balke and Wohar (2002, 2006) examine both cases 3a and 3b but all three studies assume no bubble is present.

¹³ Even for cases in which priors admitted the permanent shocks, the prior distributions of the variance of these shocks was centered around values quite small relative to that of variance of temporary shocks.

¹⁴ To estimate the prior distribution of the relative contributions, we simulated 10,000 artificial histories of the Markov Switching State Space model where for each history the parameters of the model were drawn from their prior distributions. For each artificial history, the relative contribution was defined to be ratio of the sample

some of the entries in detail. When the priors admit only one permanent component (in this case dividends) for the market fundamentals, see row 3 of Table 1, the prior distribution implies that ten percent of the time the percentage contribution of the market fundamentals to the variability of the log price dividend ratio is close to zero (1.17×10^{-1}), fifty percent of the time the percentage contribution of the market fundamentals is less than 17.89%, and 10 percent of the time market fundamentals explain more than 99.70% of log price dividend variability. On the other hand, for fifty percent of the draws from the prior distribution, the bubble explains over 79.82% of the log price dividend variability. Thus, for this case, most of the time the prior distribution implies that the bubble will have a larger contribution to log price dividend variability than will the market fundamentals. Indeed, from Table 1 one observes that the prior distribution generally places more weight on the relative contribution of the bubble than it does on the market fundamentals. This is particularly true for case 1 and case 2 which impose extremely strong prior beliefs about the absence of permanent changes in the market fundamentals. Only in the case where there are two permanent factors for the market fundamentals, does the prior distribution imply similarly sized contributions from the market fundamentals and the bubble.

We also considered a “tight” prior that the variance for period-to-period shocks to the bubble (v_t^b), or σ_b^2 , is small relative to variances of shocks to the market fundamentals.¹⁵ We considered this type of prior for two reasons: first, to emphasize the expanding and collapsing regimes aspect of the bubble; and second, to help distinguish between time varying required

variance of the market fundamental (or the bubble) relative to the sample variance of the log price dividend ratio (i.e., $100 \cdot \text{var}(p^{\text{mf}}) / \text{var}(p)$ or $100 \cdot \text{var}(b) / \text{var}(p)$). Note it is possible for the percent contribution to be greater than 100% if there is sufficient negative sample covariance between the various components.

¹⁵ Here, rather than drop the bubble variable entirely, we give “weight” to the prior distribution for σ_b^2 roughly equal to 10,000 sample observations. One can think of the posterior distribution as giving a weight of 214 to the data and a weight of 10,000 to the prior. This allows a regime switching bubble to be present but without substantial period-to-period shocks.

returns and random fluctuations in the bubble, particularly in the collapsing or nonexpanding bubble regime, $I_t = 0$. For example, if one happened to be in the non-explosive or collapsing bubble regime for the entire sample then this would be observationally equivalent to time varying required returns—neither of which are directly observable.

4. Results.

We examine quarterly data on S&P 500 stock prices and dividends.¹⁶ The sample period runs from 1952Q1 to 2005Q2. We run 50,000 iterations of the MCMC algorithm using the last 10,000 draws to estimate the joint posterior distribution.

We discuss the results for each of the four alternative priors about the market fundamentals starting with the most restrictive and ending with the most “uninformed” prior.

Case 1. Constant required returns and no persistent dividend growth factor.

Figure 1 presents the relative contribution when one’s priors allow for only a temporary dividend growth component for the market fundamentals but “uninformed” priors about the variance of the period-to-period shocks in the bubble (σ_b^2).¹⁷ Here nearly all of the movements in log price dividend ratio are explained by the bubble component; the market fundamentals contribute very little to stock price movements. By assuming constant required returns, we essentially force all market fundamentals to be reflected in expected future dividend growth. Consistent with Shiller (1981), stock price movements are much too volatile (and persistent) to be explained only by temporary fluctuations in dividend growth; to explain the log price dividend requires substantial period-to-period variability in the bubble. Table 2 displays the

¹⁶ Dividends are actually a four quarter moving average. Dividends and SP500 stock index are from the database compiled by Haver Analytics.

¹⁷ For ease of comparison we display deviations from the sample mean for the log price dividend ratio and contributions of market fundamental components and contributions of the bubble. We do this for subsequent decompositions (Figures 3, 4, 6-11) as well.

prior and posterior distributions for variances of shocks to market fundamentals and to the bubble. Looking at the case where one has “loose” priors over the variance of shocks to the bubble, we see that the posterior distribution of the variance of period-to-period shocks in the bubble shifts dramatically from its prior distribution; the variance of period-to-period shocks in the bubble increases by a factor of around 10^8 (see Table 2, case 1, row 2). In this case, the posterior distribution implies that 10 percent of the time the variance of period-to-period shocks to the bubble component will be greater than 72.66, substantially larger than under the prior.

Figure 1 suggests that the bubble contributes to nearly all the variation in the price-dividend ratio, while market fundamentals contribute very little. But the implied behavior of the bubble does not correspond to what one typically thinks as a bubble—alternating episodes of explosive and collapsing stock prices. Figure 2 panel A plots the probability of being in the expanding bubble regime. Here the ex post probability of an expanding bubble hovers around 0.5 for the entire sample. Recall that the prior probability of being in the expanding regime is also 0.5. This suggests that the data has little to say about regime switching behavior of the bubble term. As a result, most of the variability in the bubble is due to period-to-period shocks and not to periodically switching between expanding and collapsing regimes.

If one believes that regime switching or periodic episodes of expanding and contracting are what characterizes a bubble then we can emphasize that aspect of a bubble by placing a strong prior that the period-to-period shocks to the bubble are small.¹⁸ Figure 2 Panel B presents the posterior probability of being in the expanding bubble regime when a tight prior for σ_b^2 is assumed. Unlike the “uninformed” prior displayed in Panel A, here we observe substantial regime switching behavior. However, the regimes are not particularly persistent as the bubble

¹⁸ When one has tight priors that σ_b^2 is “small”, we still observe a large shift in the posterior distribution but it is not nearly as dramatic as when “loose” priors were assumed (compare rows 2 and 4 in case 1 of Table 2).

term appears frequently to jump back and forth between expanding and contracting regimes. Furthermore, the increase in stock prices in the 1990s is not reflected in a persistent switch to the expanding bubble regime during that time period.

Figure 3 presents the posterior distributions of the relative contributions of market fundamentals and bubble term for the model in which we impose a “tight” prior on the period-to-period shocks in the bubble. Once again, the market fundamentals (here only dividend growth) do not contribute much to movements in log price dividend ratio--the posterior distribution for shocks to dividend growth is very similar for the “loose” and “tight” priors on σ_b^2 . However, the bubble contribution to period-to-period fluctuations in stock prices is substantially less as well. In fact, the contribution of the bubble is mainly reflected in a smooth trend in the log price dividend ratio. The “noise” term in the log price dividend ratio contributes to the most of the variability in log price dividend ratio; that is, neither the bubble nor market fundamentals explain the log stock price dividend ratio when there is a tight prior for σ_b^2 .

Case 2. No persistent factors in the market fundamentals.

This case corresponds to where there are very strong priors that the market fundamentals are stationary. Figure 4 presents the relative contribution when we assume relatively uninformed priors about the variance of the period-to-period shocks in the bubble (σ_b^2). Here nearly all of the movements in log price dividend ratio are explained by the bubble component; the market fundamentals contribute very little to stock price movements. As in Case 1, because neither dividend growth nor realized returns show much persistence, when these fundamental factors are restricted to have no permanent components a “large” bubble component is helpful in capturing the persistence seen in the actual log price dividend ratio. Once more the posterior distribution

of the variance of period-to-period shocks in the bubble shows a dramatic increase over the prior distribution (see case 2, row 3 of Table 2). Also as in the previous case, the data appears to provide little information about the nature of regime switching in the bubble (see Figure 5, Panel A) as the ex post probability of being in the bubble regime hovers around .5.

When we start with very tight priors about σ_b^2 , limiting the period-to-period variability of bubble, the market fundamentals play a larger role (see Figure 6). In fact, unlike Case 1 above, the market fundamentals can explain some of the key episodes of stock price movements in our sample such as the decline in stock prices in the mid 1970s and the dramatic run up and subsequent decline in stock prices in the 1990s and early 2000s. The bubble contributes only to a general upward drift in the log price dividend ratio over our sample. Furthermore, the implied regime behavior (see Figure 5, Panel B) is not consistent with persistent periods of expanding and collapsing. In particular, the dramatic increase in stock prices in the 1990s was not due to being in the expanding bubble regime nor was the decline in stock prices in the 2000s due to a switch to a collapsing bubble regime. Comparing the constant required returns case above with the time-varying (but temporary) required returns case, suggests that it is required returns rather than dividend growth that enables the market fundamentals to capture much more of the variability in the log price dividend ratio when we place a tight prior on the variance of the period-to-period shocks to the bubble. This is consistent with previous variance decomposition studies that assumed dividend growth and returns to be stationary, for example Campbell and Shiller (1988), Cochrane (1992), and Campbell and Ammer (1993)).

Case 3. Only one permanent component in market fundamentals.

Figure 7 displays the posterior mean, 10th and 90th percentiles of the posterior distribution of market fundamentals and bubble contributions to stock price movements for the model in which only dividend growth has a permanent component. Recall that this corresponds to the case where one has very strong priors that there is no permanent factor in required returns. Figure 8 displays the posterior distribution for the model in which only the required return has a permanent component.

Figures 7 and 8 suggest that as long as there is at least one permanent factor in the market fundamentals—it does not matter whether the permanent component is in dividend growth or required returns—the sample data do not provide much support for the presence of a rational bubble. Recall that the priors for the variances of shocks to the various factors, except for one of the permanent components, are relatively uninformed and that, if anything, the prior distribution gives substantially more weight to the bubble component (see Table 1, rows three and four). Yet despite the tendency of the prior distribution to favor the bubble, the posterior distribution admits virtually no contribution of the bubble to movements in the log price dividend ratio. In fact, the posterior distribution of the indicator variable for the bubble suggests that the probability of a bubble being in the expanding regime is essentially zero for the entire sample. This result is robust to tightening the prior distribution for the variance of period-to-period bubble shocks (not surprisingly) and to tightening the prior distribution of the variance of the noise terms.

Figure 9, Panel A, displays the permanent factor for dividend growth implied by prior 3a while Figure 9, Panel B, displays the permanent factor for required returns for prior 3b. The estimated permanent factors for both dividend growth and required returns are smooth compared to the actual series and the posterior distributions for these variables are quite tight. This

suggests that it does not take a large permanent factor in either dividends or required returns for market fundamentals to explain stock price movements.

Case 4. Market fundamentals include permanent factors for dividend growth and required returns.

Here we consider relatively uninformed priors for the variances of shocks to all the unobserved factors including the bubble. Figure 10 displays the mean and 10th and 90th percentiles of the posterior distribution of contributions to log price dividend fluctuations. From the figure, we can infer log price dividend movements are driven almost entirely by market fundamentals. As in case 3, the posterior distribution of the indicator variable for the bubble suggests that the probability of bubble being in the expanding regime is essentially zero for the entire sample. Also, as in case 3, the posterior distribution is robust to tightening the prior distribution for the variance of period-to-period bubble shocks and to tightening the prior distribution of the variance of the noise terms.

Figure 11 displays the mean, 10th and 90th percentiles of the posterior distribution for the permanent components of dividends and required returns. The mean of the posterior distribution of the permanent component of dividend growth tracks actual dividend growth quite closely, except for the very high frequency movements in dividend growth. However, the posterior distribution is quite diffuse with the interior 80 percent of the posterior distribution spanning a wide range of values for permanent dividend growth. For required returns, actual realized returns are substantially more volatile than the mean of the posterior distribution for the permanent factor of required returns, although, here too, the interior 80 percent of the posterior distribution spans a wide range of values. That the data do not provide precise information about

the relative sizes of the permanent factors in dividend growth and required returns is not surprising given that most of the information about low frequency movements in dividend growth or required returns would actually be in stock prices rather than the market fundamental series themselves.¹⁹

5. Robustness

In this section, we consider how sensitive our results are to the use of the Campbell-Shiller log linear approximation, the specification of the “persistent” components of the market fundamentals, the use of earnings instead of dividends as a measure of cash flows, and finally the use of NASDAQ stock index instead of the S&P 500 stock index. In sum, the previous results are remarkably robust to these alternative specifications and data.²⁰

5.1 Campbell-Shiller Log Approximation.

In the above analysis, we used the Campbell-Shiller log linear approximation to derive the observation equation linking the log price dividend ratio to the market fundamentals. As we pointed out above, the log linear approximation provides a very convenient framework empirically examining stock price determination in an environment where required returns vary over time. The drawback is that our analysis literally examines a bubble in the log price dividend ratio rather in stock prices directly.

¹⁹ We have made this point previously in Balke and Wohar (2002, 2006).

²⁰ We examined several sets of alternative priors along with those discussed in the text. One was to have very strong priors that the intercept in the bubble (\bar{b}) was positive. This would make it more likely that the bubble was positive. The other was to start with a prior distribution for σ_b^2 that was centered around the same value as that of the variance of the permanent components and the noise. Neither of these alternative priors changes our major findings.

How would a bubble in stock prices be reflected in the log price dividend ratio? To get at this question, we take a simple Gordon model where dividend growth and required returns are constant over time and add a bubble to it. The resulting price-dividend ratio is given by:

$$P_t / D_t = (1 + d)/(r - d) + B_t / D_t$$

where P_t is the stock price, D_t is the level of dividends, B_t is the bubble term, d is the constant dividend growth rate and r is the required rate of return. For a bubble to be present, it must satisfy $E_t B_{t+1} = (1 + r)B_t$. Figure 12 displays the hypothetical path of the log price dividend ratio in the presence of a (non-bursting) bubble. For comparison, we also plot what the hypothetical log price dividend ratio would be for a bubble in the Campbell-Shiller approximation (recall the bubble must satisfy equation 6). We calibrate the parameters in these examples to be roughly consistent with actual data over our sample: $r = .025$ (10 percent nominal annual returns), $d = .0125$ (5 percent annual nominal dividend growth), average $\log(\text{price/dividend})$ over our sample of 4.84, and $\rho = 0.992$. The starting value of the bubble for the two cases was chosen so that the average of log price dividend ratio over the sample was equal to the historical average over our empirical sample. As we can see, a bubble in the Campbell-Shiller approximation implies a slightly greater appreciation in the log price dividend ratio than the standard bubble, yet the overall behavior of the Campbell-Shiller bubble is quite similar. This suggests in finite samples using the Campbell-Shiller approximation can capture an actual bubble reasonably well.²¹

The parameter, ρ , in the Campbell-Shiller approximation plays an important role in our analysis as it determines in part the factor loading for the persistent components of the market

²¹ As the time horizon lengthens, for a fixed value of ρ the two bubble variables will diverge. In fact for the Gordon model bubble, the log price dividend ratio exhibits a near linear trend as the bubble gets larger. Of course if the value of ρ also changes as the sample gets larger, then the two bubble components will exhibit similar long-run properties.

fundamentals in the log price dividend equation. To see whether our results are sensitive to alternative, plausible values of ρ , we examined the case where

$\rho = \exp(p_{\min}) / (1 + \exp(p_{\min}))$ where p_{\min} is the minimum log price dividend ratio over our sample and $\rho = \exp(p_{\max}) / (1 + \exp(p_{\max}))$ where p_{\max} is the maximum log price dividend ratio over our sample. Regardless of which value of ρ used, the qualitative nature of the results do not change: once one allows a persistent component in one or both of the market fundamentals the bubble component plays no role in explaining movements in the price-dividend ratio.

5.2 Persistent rather than permanent components in market fundamentals.

Our specification of the market fundamentals allowed for two components for both dividend growth and required returns: a persistent and a temporary component. Above, we modeled the persistent component as a random walk. One might question the presence of a permanent component in dividend growth and required returns, although we argue above and elsewhere (Balke and Wohar (2002, 2006)) that the data cannot rule out the presence of small permanent components for these series. To determine whether our results hinge on the assumption of a permanent component rather than just on a specification of a “persistent” component, we examined a state space model where we replaced the random walk permanent component with a “persistent” but stationary component of dividends and/or required returns. We specified the persistent component as being an AR(1) with an autoregressive coefficient of .99. This model implies that the log price dividend ratio is stationary in the absence of a bubble as both dividend growth and required returns are themselves stationary.

Posterior distributions of decompositions of log price dividend ratio are qualitatively similar to those presented above (compare Figure 13 with Figure 11). The market fundamentals,

so long as they include “persistent” component(s) for dividend growth or (and) required returns, are capable of explaining nearly all the fluctuations in the log price dividend. Thus, it is not the permanent nature of dividend growth or required returns per se that matters but the persistence. The fluctuations in the implied “persistent” components, however, are quantitatively larger than those for the model with permanent components (see Figure 14). The reason is that the factor loadings for “persistent” components are nearly half as large as the factor loadings for the permanent components. Thus, in order to explain fluctuations in log price dividend, fluctuations in the “persistent” components must be roughly twice as large.

5.3 Earnings growth and price earnings ratio

Because dividend growth may not reflect all the cash flow generated by stock, we consider how sensitive our results are if we replace dividends by earnings. Applying the Campbell-Shiller approximation to the price earnings ratio rather than the price dividend ratio and using earnings growth rather than dividend growth, we reran our analysis. Qualitatively the results for log price earnings ratio are similar to those for the log price dividend ratio.

Rather than display the full range of cases examined above, we present the arguably two most interesting cases. Figure 15 displays the contribution of the market fundamentals and bubble to fluctuations in the log price earnings ratio when the prior distribution allows for permanent factors in both earnings growth and required returns while Figure 16 displays the contributions when priors rule out permanent components for earnings growth and required returns and period-to-period shocks in the bubble are “small”. As in the case for dividends, the market fundamentals are capable of explaining all the fluctuations in log price earnings ratio if one allows for the possibility of permanent factors in earnings growth and/or required returns. If,

however, one rules out the possibility of permanent factors in the market fundamentals the bubble can play a larger role. If one forces the period-to-period shocks to be small, the bubble explains the drift upward in the price earnings ratio over the sample but a substantial portion of the increase in the price earnings ratio in the 1990s is still due to market fundamentals.

5.4 NASDAQ Stock Price Index

Given the discussion of a “bubble” in internet and high technology stocks (e.g, Cunado, Gil-Alana, and Perez de Garcia, 2005), it would be interesting to apply our analysis to a stock index that placed greater weight on these types of stock than the S&P 500. Thus, we conduct our analysis on the value weighted index of the NASDAQ from the CRSP database. Again we use quarterly data and our sample here runs from 1974:1 to 2003:4.²²

Once again, the previous results for the S&P 500 index hold qualitatively for the NASDAQ index. Figures 17-20 reproduce some of the results for the NASDAQ index. From Figure 17, we observe that as before once one allows for permanent factors in the market fundamentals, the market fundamentals explain nearly all the variation in the log price dividend ratio for the NASDAQ while the bubble explains nearly none of the variation. This is despite the dramatic run up in the NASDAQ over our sample. One reason why the market fundamentals model can easily explain the dramatic trend in stock prices is that the Campbell-Shiller approximation for the NASDAQ implies a value of $\rho = 0.997$ which in turn implies factor loadings on the permanent components of the market fundamentals of close to 326 in the log price dividend equation. Thus, very small changes in the permanent factors are capable of

²² As with the S&P500, we use a four quarter moving average of paid dividends to remove the strong seasonal pattern in dividend payments.

explaining the upward drift of the log price dividend ratio over the sample. Figure 18 shows that with the NASDAQ, the data have difficulty determining which market fundamental, dividend growth or required returns, contributes more to log price dividend movements—the interior 80% band of the posterior distribution implies a large number of alternative permanent dividend growth or required return components.

Figure 19 displays the implied contribution of the market fundamentals and bubble when no permanent components for the market fundamentals are present and when the period-to-period shocks in the bubble are small. As before, while the bubble is capable of capturing the general upward drift in the log price dividend ratio for the NASDAQ, it still leaves major swings in the log price dividend ratio explained by changes in the market fundamentals (here primarily required returns). Finally, Figure 20 displays the estimated probability of being in the explosive regime during the sample. Unlike the results for the S&P 500, for the NASDAQ we observe less switching back and forth from expanding to nonexpanding regimes suggesting more persistence in the expanding regime for the NASDAQ. The generally steady increase in the log price dividend ratio for the NASDAQ over our sample requires more persistence in the expanding regime than does the S&P 500.

6. Discussion.

Why do the priors matter so much? The standard answer would be that data is not very informative about the determinants of stock prices. While the standard answer is literally true, the data does tell us something about what features of the market fundamentals and bubble are crucial for explaining stock prices movements. The log price dividend ratio is very persistent especially when compared to dividend growth or actual returns. To capture the persistence in

the log price dividend ratio, one or more the components of the log price dividend ratio must also be persistent. When prior beliefs force dividend growth and required returns to be comprised of a single temporary component (plus noise for dividend growth) it is very difficult for the market fundamentals to generate the type of persistence seen in the log price dividend ratio. This opens the door for the bubble component to explain stock price movements. However, the implied bubble component does not look like the expanding/bursting bubble typically thought of in the popular press or in the academic literature.

Once we relax the very strong prior beliefs that there are no persistent components in dividend growth and required returns then the market fundamentals are capable of explaining both the variability and persistence of the log price dividend ratio; the bubble is not needed. Indeed, not only is a bubble not needed but the properties implied by the rational bubble do not appear to be supported by the data and, thus, the posterior distribution attributes virtually no contribution to the bubble. This contrasts with the contribution of the bubble implied by the prior distribution (recall Table 1). Thus, here the data is indeed informative and moves the posterior distribution away from a significant contribution by the bubble.

Note that the implied behavior of the market fundamental components are not out-of-line with actual dividend growth or realized returns even though these series do not appear to be very persistent—the implied permanent or persistent component is quite small and would be dominated in the data by the temporary fluctuations in these variables. Thus, with the exception of the log price dividend ratio itself, the data is not very informative about small persistent components in both dividend growth and required returns. In fact, when the prior beliefs allow persistent components in both dividend growth and required returns, there are many alternative persistent components that appear to be consistent with the data, hence disperse posterior

distributions about the persistent factors displayed in Figures 11, 14, and 18—the data cannot determine whether it is persistent changes in dividend growth, persistent changes in required returns, or both.

What about other types of deviations from market fundamentals? It is possible that either our bubble component and/or the unobserved required returns factors are in fact picking up deviations from market fundamentals not associated with a rational bubble.²³ To the extent that these alternative explanations can put restrictions on the dynamics on stock prices (or log price dividend ratio) then we can evaluate them empirically, otherwise we cannot distinguish them from unobserved required returns. At least a rational bubble imposes fairly strong restrictions on the dynamic non-market fundamental component and these restrictions can help us to identify a bubble in the data.

7. Conclusion

The objective of this paper is to qualitatively and quantitatively investigate how important the bubble component in the S&P 500 stock price index is relative to market fundamentals. We attempt to gain some insight into the relative size of the bubble component and how much it contributes to fluctuations in the log price dividend ratio. We find that different priors concerning market fundamentals and bubble components affect the inference made about the relative contribution of market fundamentals and rational bubbles to stock price fluctuations. If one's priors allow for a permanent component in dividend growth and/or required returns (albeit quite small), then the posterior distribution suggests that market fundamentals are capable of explaining nearly all of the fluctuations in the log price dividend ratio. When both dividend growth and required returns

²³ Models incorporating fads, momentum, noise traders are examples of nonfundamental sources of variability; see for example, De Long, et al. (1990a,b) and Shleifer (2000).

are allowed to have permanent components, then the posterior distribution of the permanent components of these variables are quite diffuse suggesting that the data has difficulty identifying which of these two factors is responsible for stock price movements. If, on the other hand, one's priors restrict the market fundamentals to have no permanent components then the bubble component can be substantially more important. However, the implied behavior of the bubble does not appear to exhibit switching between persistent periods of expanding and collapsing. In particular, the run-up and decline in stock prices in the 1990s and 2000s cannot be said to have been caused by a switch from an expanding to collapsing bubble regime. These results are robust to replacing the permanent components of the market fundamentals with only "persistent" components, to replacing dividends with earnings, and to using the NASDAQ stock index instead of the S&P 500 Index.

As we suggested above, the key puzzle for a market fundamentals explanation is the apparent persistence in the log price dividend ratio relative to dividend growth and realized returns. Dividend growth and realized returns are dominated by relatively high frequency movements and as such may be noisy signals about small, low frequency movements in these variables. To sort out whether the data supports a small but permanent component in dividend growth or required returns, we must bring additional information to bear on this decomposition. In particular, macro data on the dynamics of output, consumption, investment as well as the behavior of other asset prices may shed light on the income stream and discount factors for stocks. This may allow the data to speak more decisively about the relative contribution of the market fundamentals and a bubble.

Appendix

We estimate the model by Bayesian MCMC methods. We want the posterior distribution for both the parameters, Θ , and the unobserved state vector, \mathbf{S} , given the data, \mathbf{Y} , or $P(\Theta, \mathbf{S} | \mathbf{Y})$. Unfortunately, the posterior distribution is not easy to derive. However, with standard conjugate priors, the posterior distribution of parameters given \mathbf{S} , $P(\Theta | \mathbf{S}, \mathbf{Y})$, are relatively straightforward to derive. Similarly, given the parameters, finding posterior distribution of unobserved factors, $P(\mathbf{S} | \Theta, \mathbf{Y})$, is relatively straightforward to calculate. Starting from arbitrary initial values for $\mathbf{S}^{(0)}$ and $\Theta^{(0)}$, one samples $\mathbf{S}^{(1)}$ from $P(\mathbf{S} | \Theta^{(0)}, \mathbf{Y})$. Then given $\mathbf{S}^{(1)}$, sample $\Theta^{(1)}$ from $P(\Theta | \mathbf{S}^{(1)}, \mathbf{Y})$. Continue recursively sampling $\mathbf{S}^{(i)}$ from $P(\mathbf{S} | \Theta^{(i-1)}, \mathbf{Y})$ and $\Theta^{(i)}$ from $P(\Theta | \mathbf{S}^{(i)}, \mathbf{Y})$. The resulting sample distribution of $(\mathbf{S}^{(i)}, \Theta^{(i)})$ converges to $P(\Theta, \mathbf{S} | \mathbf{Y})$. In conducting our repeated sampling, we actually employ a combination of Gibbs sampling and the Metropolis-Hasting algorithm to construct an estimate of the posterior distribution.

The specific steps of MCMC algorithm are as follows:

1. Given $Y_t, \mathbf{R}, \mathbf{Q}_m, \sigma_B^2, \gamma, \bar{\mathbf{b}}, \mathbf{F}_m, \mathbf{I}_t, s, q$ we can find the conditional distribution of S_t (see Kim and Nelson (1999)). Draw realization of $S_t^{(i)}$ from this conditional distribution.
2. Given $\sigma_B^2, \gamma, \bar{\mathbf{b}}, b_t, s, q$ we one can find the conditional distribution of I_t (see Kim and Nelson (1999)). Draw realization of $I_t^{(i)}$ from this distribution.

3. Given $\mathbf{R}, \mathbf{Q}_f, \mathbf{S}_t^f = (d_t^p, r_t^p, d_t^\tau, r_t^\tau, d_{t-1}^\tau, r_{t-1}^\tau, \dots, d_{t-m+1}^\tau, r_{t-m+1}^\tau)'$, \mathbf{Y}_t calculate the conditional distribution for $\Theta_1, \dots, \Theta_m$. Note, we use the Metropolis-Hastings algorithm as \mathbf{H} is a nonlinear function of Θ_i 's.

$$(A1) \quad \mathbf{Y}_t = \mathbf{A} + \mathbf{H}(\Theta)\mathbf{S}_t + \mathbf{W}_t.$$

$$(A2) \quad \mathbf{S}_t^{f*} = \mathbf{F}^*(\Theta)\mathbf{S}_{t-1}^f + \mathbf{v}_t^f,$$

with $\mathbf{S}_t^{f*} = (d_t^p, r_t^p, d_t^\tau, r_t^\tau)'$ and $\mathbf{F}^*(\Theta)$ is the first four rows of the $\mathbf{F}(\Theta)$ matrix. Take a candidate draw, $\tilde{\Theta}$ from the posterior distribution of the linear regression implied by equation (A2); draws of $\tilde{\Theta}$ in which roots of $\mathbf{F}(\Theta)$ are inside the unit circle are rejected and redrawn. Given $\tilde{\Theta}$, set

$$\Theta^{(i)} = \tilde{\Theta} \text{ with probability } \alpha = \min\left(\frac{f(\mathbf{Y} | \tilde{\Theta}, \mathbf{S})}{f(\mathbf{Y} | \Theta^{(i-1)}, \mathbf{S})}, 1\right) \text{ where } f(\mathbf{Y} | \Theta, \mathbf{S}) \text{ is the sample likelihood}$$

implied by equation (A1) given the parameter vector Θ , and the realizations of the state variables \mathbf{S} .

4. Given \mathbf{I}_t , one can find the conditional distribution of s and q .

For prior distributions of $s \sim \text{beta}(\delta_{11}^{s0}, \delta_{01}^{s0})$ and $q \sim \text{beta}(\delta_{00}^{q0}, \delta_{10}^{q0})$, the posterior distributions are:

$s \sim \text{beta}(\delta_{11}^{s0} + n_{11}, \delta_{01}^{s0} + n_{01})$ and $q \sim \text{beta}(\delta_{00}^{q0} + n_{00}, \delta_{10}^{q0} + n_{10})$ where n_{ij} is the number of

observations in which $(\mathbf{I}_t = i, \mathbf{I}_{t-1} = j)$. We set prior $\delta_{11}^{s0} = \delta_{00}^{q0} = 5 \times .95$ and $\delta_{01}^{s0} = \delta_{10}^{q0} = 5 \times .05$.

5. Given $b_t, \mathbf{I}_t, s, q, b_t \sigma_b^2$ one can find the conditional distribution of γ, \bar{b} . If the values of

$b_t, \mathbf{I}_t, s, q, b_t \sigma_b^2$ were known, the parameters \bar{b} and γ can be obtained from the simple linear

regression:

$$\tilde{b}_t = \bar{b}x_{1,t} + \gamma x_{2,t} + v_t^b,$$

where

$$\tilde{\mathbf{b}}_t = \mathbf{b}_t - \frac{1}{1-s} \frac{1}{\rho} \mathbf{I}_t (1 - \mathbf{I}_{t-1}) - \frac{1}{q} \frac{1}{\rho} \mathbf{I}_t \mathbf{I}_{t-1}$$

$$\mathbf{x}_{1,t} = (1 - \mathbf{I}_t) - \frac{s}{1-s} \mathbf{I}_t (1 - \mathbf{I}_{t-1}) - \frac{(1-q)}{q} \mathbf{I}_t \mathbf{I}_{t-1}$$

$$\mathbf{x}_{2,t} = [(1 - \mathbf{I}_t) - s \mathbf{I}_t (1 - \mathbf{I}_{t-1}) - (1-q) \mathbf{I}_t \mathbf{I}_{t-1}] \mathbf{b}_{t-1}.$$

Define $\mathbf{B} = (\tilde{\mathbf{b}}_1, \dots, \tilde{\mathbf{b}}_T)'$ and $\mathbf{X} = \begin{pmatrix} \mathbf{x}_{11} & \mathbf{x}_{21} \\ \vdots & \vdots \\ \mathbf{x}_{1T} & \mathbf{x}_{2T} \end{pmatrix}$. Given prior distribution for $\bar{\mathbf{b}}$ and γ of

$$\begin{pmatrix} \bar{\mathbf{b}} \\ \gamma \end{pmatrix}_0 \sim \mathbf{N}(\mathbf{M}_0, \mathbf{V}_0), \text{ the posterior distribution is } \begin{pmatrix} \bar{\mathbf{b}} \\ \gamma \end{pmatrix} \sim \mathbf{N}(\mathbf{M}_1, \mathbf{V}_1)$$

where $\mathbf{V}_1 = (\mathbf{V}_0^{-1} + (\sigma_b^2 (\mathbf{X}'\mathbf{X})^{-1})^{-1})^{-1}$ and $\mathbf{M}_1 = \mathbf{V}_1 (\mathbf{V}_0^{-1} \mathbf{M}_0 + \mathbf{V}_1^{-1} (\mathbf{X}'\mathbf{X})^{-1} (\mathbf{X}'\mathbf{B}))$.

6. Given the variables $\mathbf{Y}_t, \mathbf{I}_t, \mathbf{S}_t$ and the parameter matrices $\mathbf{A}, \mathbf{H}, \gamma, \bar{\mathbf{b}}, \mathbf{F}$, we can find the conditional distributions of \mathbf{R}, \mathbf{Q}_f , and σ_B^2 respectively. Assume a prior distribution for \mathbf{R} of an inverse-wishart (IW), then the posterior distribution of \mathbf{R} is given by

$\mathbf{R} \sim \text{IW}(v_{R0} + T, v_{R0} \times \mathbf{R}_0 + T \times \hat{\mathbf{R}})$ where v_{R0} and \mathbf{R}_0 are parameters from the prior distribution,

T is the number of sample observations, and $\hat{\mathbf{R}} = \sum_{t=1}^T \mathbf{W}_t \mathbf{W}_t' / T$ with $\mathbf{W}_t = \mathbf{Y}_t - \mathbf{A} - \mathbf{H}\mathbf{S}_t$.

Similarly, given standard conjugate priors, $\mathbf{Q}_f \sim \text{IW}(v_{Q0} + T, v_{Q0} \times \mathbf{Q}_{f0} + T \times \hat{\mathbf{Q}}_f)$ where v_{Q0} and

\mathbf{Q}_{f0} are parameters from the prior distribution, and $\hat{\mathbf{Q}}_f = \sum_{t=1}^T \mathbf{v}_t^f \mathbf{v}_t^{f'} / T$ with $\mathbf{v}_t^f = \mathbf{S}_t^{f*} - \mathbf{F}^* \mathbf{S}_{t-1}^f$.

Finally, $\sigma_b^2 \sim \text{II}(v_{\sigma0} + T, v_{\sigma0} \times \sigma_{b0}^2 + T \times \hat{\sigma}_b^2)$ with $\hat{\sigma}_b^2 = \sum_{t=1}^T (v_t^b)^2 / T$ and $v_t^b = \tilde{\mathbf{b}}_t - \bar{\mathbf{b}} \mathbf{x}_{1,t} - \gamma \mathbf{x}_{2,t}$.

Priors:

$\theta_{d,i} \sim N(0,2)_{I(|\lambda(\theta_d(L))|<1)}$ (only keep draws where the roots of lag polynomial are outside the unit circle).

$\theta_{r,i} \sim N(0,2)_{I(|\lambda(\theta_r(L))|<1)}$ (only keep draws where the roots of lag polynomial are outside the unit circle).

$\gamma \sim N(0,2)_{I(0<\gamma<1)}$ (only keep draws where $0 < \gamma < 1$).

$\bar{b} \sim N(0,1000)$

$A_p \sim N(\hat{\mu}_p, 1000)$ where $\hat{\mu}_p$ is the sample mean of the log price dividend ratio.

For the initial conditions of the unobserved states:

$b_0 \sim N(0,1000)$, $d_0^p, r_0^p \sim N(0,100)$, $\begin{pmatrix} d_0^\tau \\ r_0^\tau \end{pmatrix} | \Theta, Q_f \sim N(0, D)$, $\text{vec}(D) = (I - F_s \otimes F_s)^{-1} \text{vec}(Q_f)$

$$F_s = \begin{pmatrix} \Theta_1 & \cdots & \Theta_m \\ I_{2(m-1) \times 2(m-1)} & & 0_{2(m-1) \times 2} \end{pmatrix}.$$

The prior distributions for the variances are given by:

$Q_f \sim IW(v_{Q_0}, v_{Q_0} \times Q_{f_0})$ where IW denotes inverse Wishart distribution and

$$Q_{f_0} = \text{diag}(.0001 \quad .0001 \quad 1.0 \quad 1.0).$$

$R \sim IW(v_{R_0}, v_{R_0} \times R_0)$ where $R_0 = \text{diag}(0.0001 \quad 0.0001)$.

$\sigma_b^2 \sim \Pi(v_{\sigma_0}, v_{\sigma_0} \times \sigma_{b_0}^2)$ where $\sigma_{b_0}^2 = (0.0000001)$.

For the case where relatively uninformative priors we set $v_{Q_0}, v_{R_0}, v_{\sigma_0} = 5$ while for “tight”

priors we set $v_{R_0}, v_{\sigma_0} = 10000$. One way to interpret these priors is given the sample size of 214

observations an “uninformative” prior contributes roughly the equivalent of five sample

observations to the posterior while a tight prior contributes roughly the equivalent 10,000 sample

observations to the posterior. For the probability of switching from bubble regimes, the prior distributions were set as: $s \sim \text{beta}(\delta_{11}^{s0}, \delta_{01}^{s0})$ and $q \sim \text{beta}(\delta_{00}^{q0}, \delta_{10}^{q0})$ were $\delta_{11}^{s0} = \delta_{00}^{q0} = 5 \times .95$ and $\delta_{01}^{s0} = \delta_{10}^{q0} = 5 \times .05$. Again, one can think of the prior as contributing a weight of roughly five sample observations to the posterior distribution. Note that given the symmetry in priors, the prior probability of being in the expanding bubble regime is 0.5.

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Table 1.
Relative contributions of market fundamentals and bubble to fluctuations in $\log(p/d)$ implied by the prior distributions of the parameters in the model.

	Percent contribution of market fundamentals			Percent contribution of bubble		
	10 th percentile	50 th percentile	90 th percentile	10 th percentile	50 th percentile	90 th percentile
Prior:						
Case 1: constant required returns and temporary dividend growth	7.40×10^{-5}	7.41×10^{-5}	7.44×10^{-5}	99.22	99.27	99.32
Case 2: Temporary dividend growth and required returns	4.82×10^{-3}	1.23	96.46	3.71	98.38	99.68
Case 3: One permanent factor						
a. Permanent dividend growth factor	1.17×10^{-1}	17.89	99.70	4.16×10^{-1}	79.82	107.72
b. Permanent required returns factor	1.20×10^{-1}	17.86	99.66	4.40×10^{-1}	80.17	107.46
Case 4: Permanent factors in both dividend growth and required returns	4.69×10^{-1}	43.87	100.78	1.30×10^{-1}	53.16	106.54

Table 2
Percentiles of prior and posterior distributions for variances of shocks to bubble and market fundamentals.

Case 1: Constant required returns and temporary dividend growth		Prior distribution (percentiles)			Posterior distribution (percentiles)		
Prior:	Variance of shock to:	10 th	50 th	90 th	10 th	50 th	90 th
“loose” prior on σ_b^2	Temp. dividend growth	0.54	1.14	3.03	2.28	2.58	2.92
	Bubble (σ_b^2)	.53x10 ⁻⁷	1.1x10 ⁻⁷	3.1x10 ⁻⁷	56.24	63.82	72.66
“tight” prior on σ_b^2	Temp. dividend growth	0.55	1.15	3.09	2.29	2.58	2.92
	Bubble (σ_b^2)	.98x10 ⁻⁷	1.0x10 ⁻⁷	1.0x10 ⁻⁷	4.6x10 ⁻⁴	2.5x10 ⁻³	9.5x10 ⁻³
Case 2: Temporary dividend growth and required returns		Prior distribution (percentiles)			Posterior distribution (percentiles)		
Prior:	Variance of shock to:	10 th	50 th	90 th	10 th	50 th	90 th
“loose” prior on σ_b^2	Temp. dividend growth	0.65	1.47	4.64	3.33	4.37	6.96
	Temp. required returns	0.64	1.49	4.78	23.13	44.47	111.3
	Bubble (σ_b^2)	.54x10 ⁻⁷	1.1x10 ⁻⁷	3.0x10 ⁻⁷	52.38	61.38	71.03
“tight” prior on σ_b^2	Temp. dividend growth	0.65	1.51	4.67	38.78	45.11	53.82
	Temp. required returns	0.65	1.49	4.67	84.40	98.29	116.6
	Bubble (σ_b^2)	.98x10 ⁻⁷	1.0x10 ⁻⁷	1.0x10 ⁻⁷	1.0x10 ⁻⁵	5.16x10 ⁻⁴	2.1x10 ⁻³

Figure 1. Posterior Distribution of the Contribution of Market Fundamentals and Bubble to Log Price-Dividend Ratio: Model with constant required returns, stationary dividend growth, and “uninformed” prior on variance of period-to-period shocks to bubble (σ_b^2).

Sample means removed.

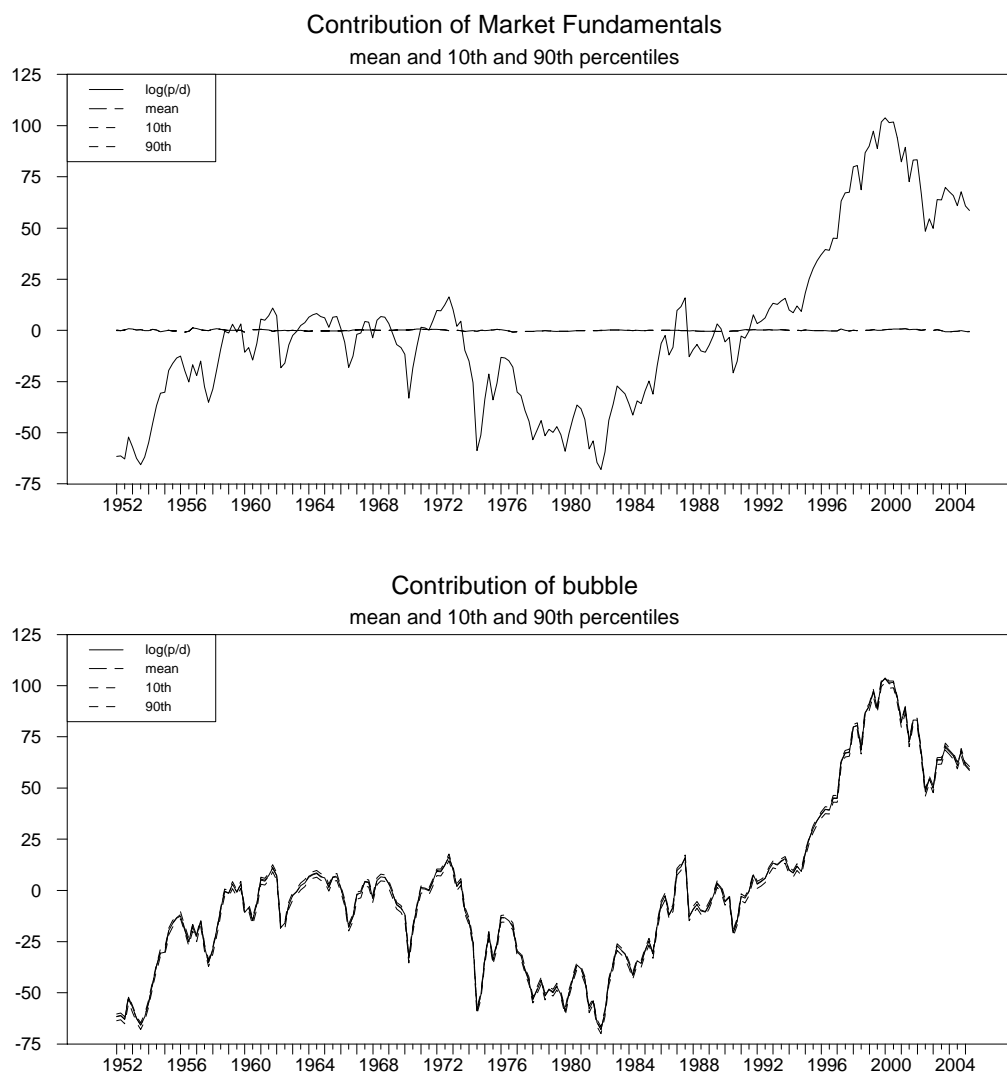
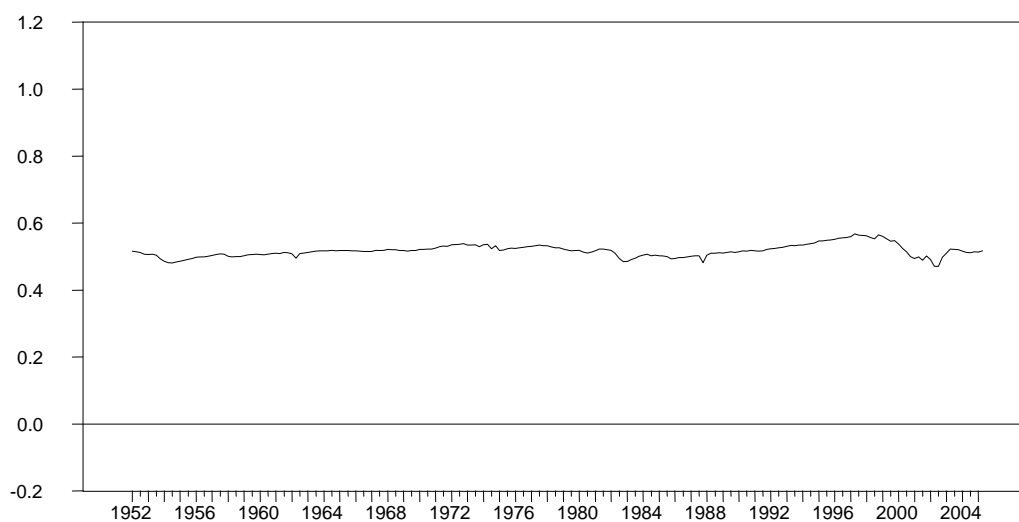


Figure 2. Probability of being in explosive bubble regime.

Panel A. Model with constant required returns, stationary dividend growth, and “uninformed” prior on variance of period-to-period shocks to bubble (σ_b^2).



Panel B. Model with constant required returns, stationary dividend growth, and “strong” prior belief that the variance of period-to-period shocks to bubble (σ_b^2) is “small”.

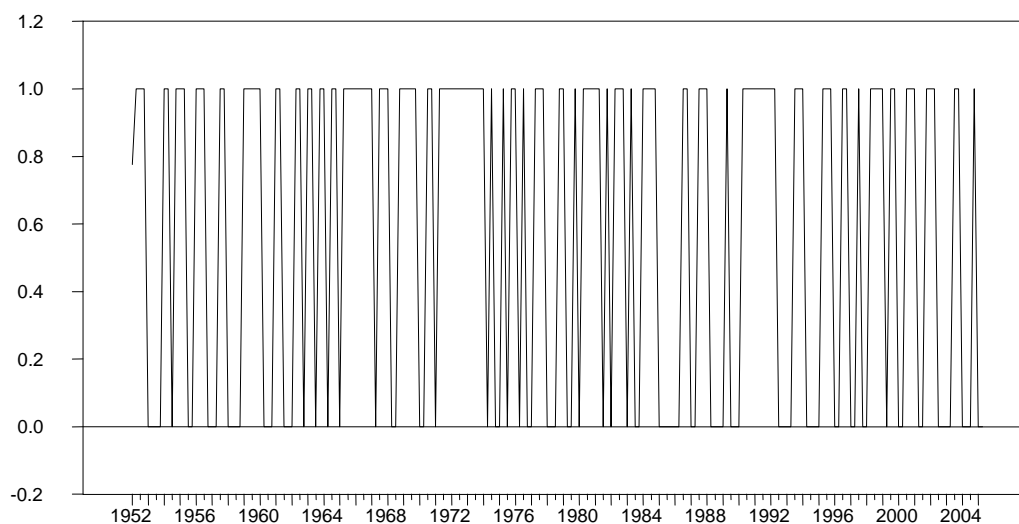


Figure 3. Posterior Distribution of Contribution of Market Fundamentals and Bubble to Log Price-Dividend Ratio: Model with constant required returns, stationary dividend growth, and “strong” prior belief that the variance of period-to-period shocks to bubble (σ_b^2) is “small”.

Sample means removed.

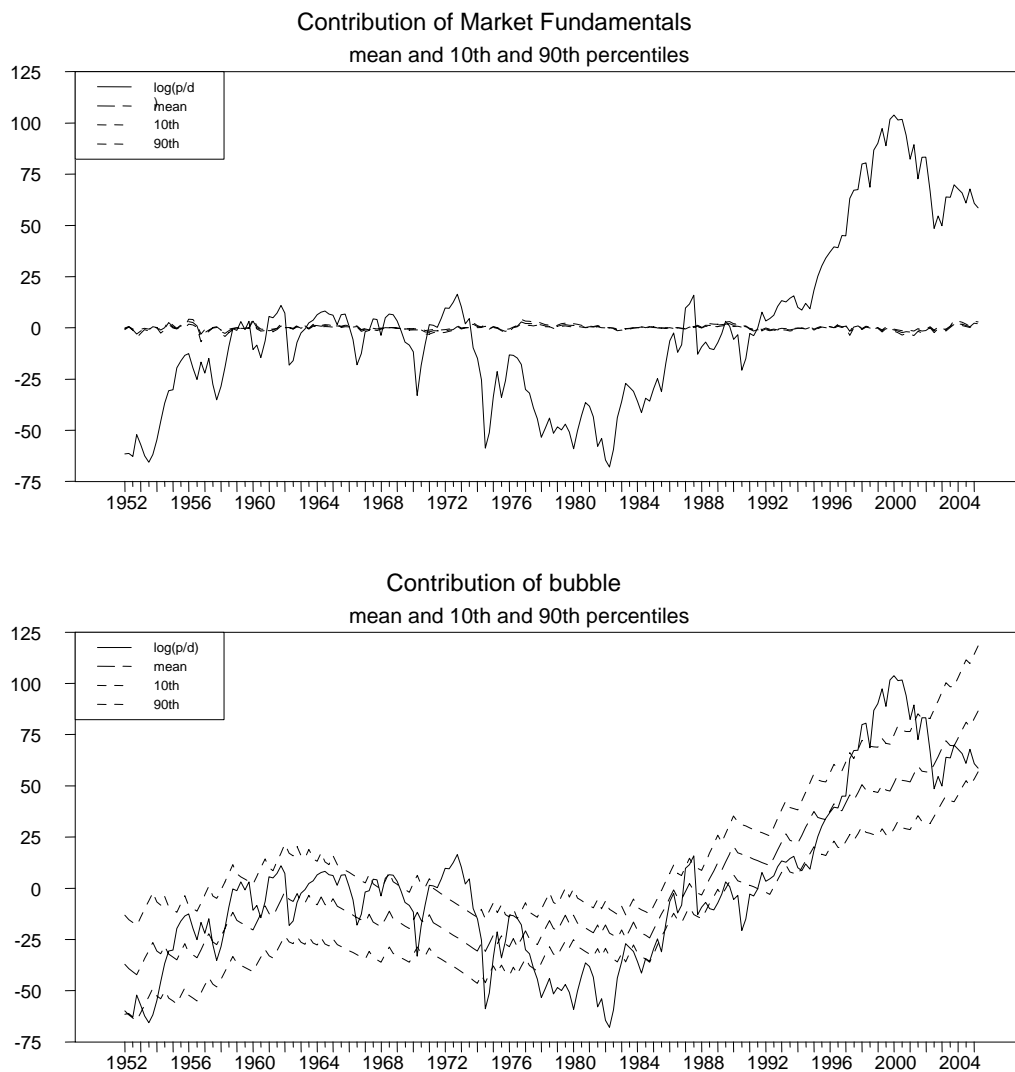


Figure 4. Posterior Distribution of Contribution of Market Fundamentals and Bubble to Log Price-Dividend Ratio: Model with no permanent component for market fundamentals and “uninformed” prior on variance of period-to-period shocks to bubble (σ_b^2).

Sample means removed.

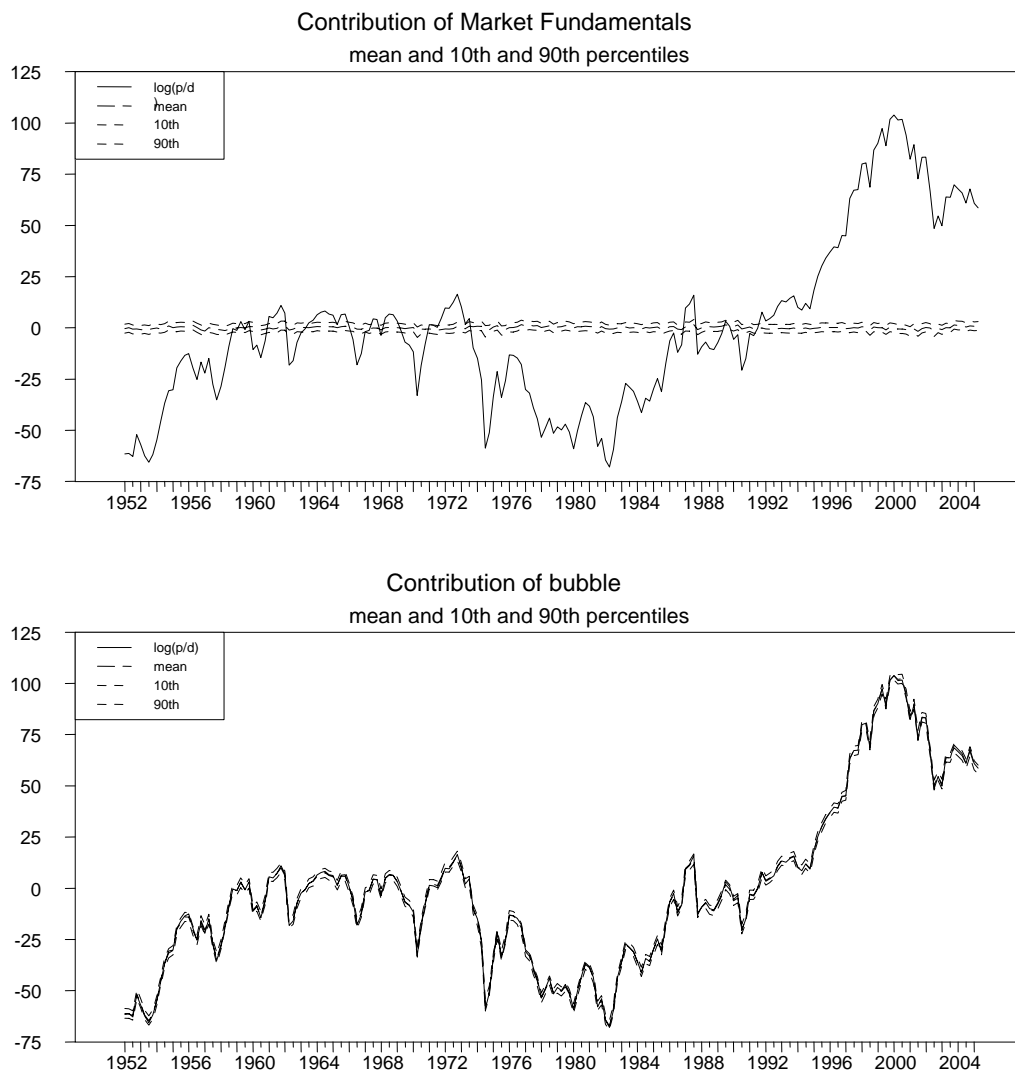
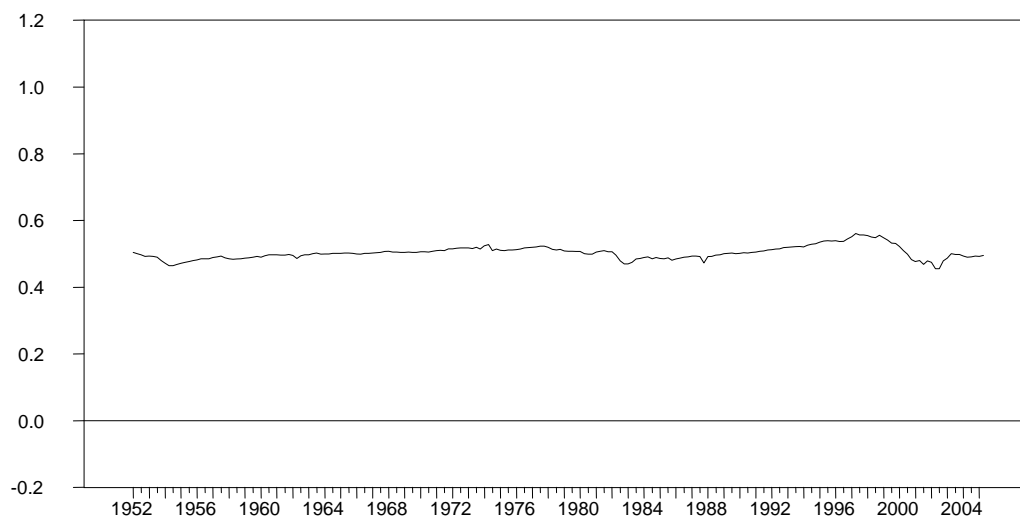


Figure 5. Probability of being in explosive bubble regime.

Panel A. Model with no persistent market fundamental components and “uninformed” prior on variance of period-to-period shocks to bubble (σ_b^2).



Panel B. Model with no persistent market fundamental components and “strong” prior that the variance of period-to-period shocks to bubble (σ_b^2) is “small”.

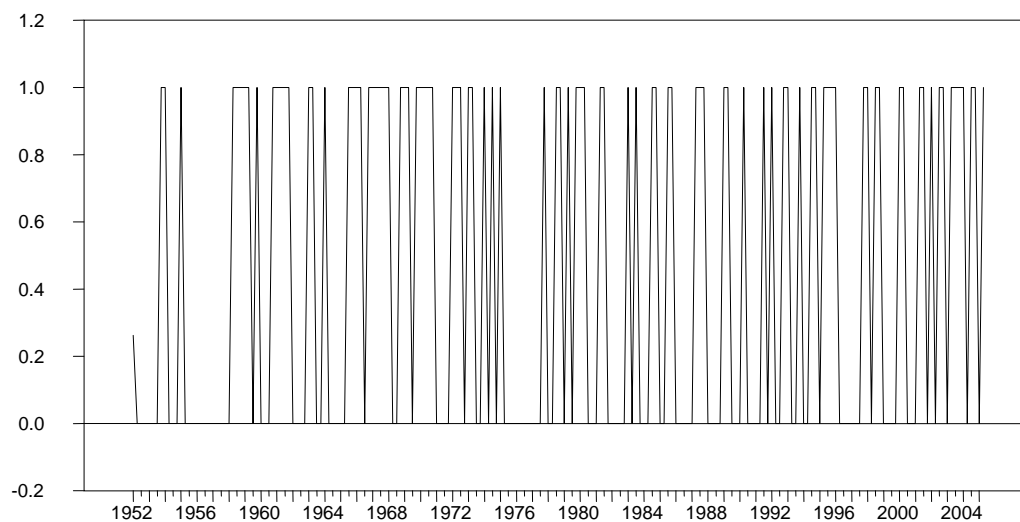


Figure 6. Posterior Distribution of Contribution of Market Fundamentals and Bubble to Log Price-Dividend Ratio: Model with no persistent component for market fundamentals and “strong” prior belief that the variance of period-to-period shocks to bubble (σ_b^2) is “small”.

Sample means removed.

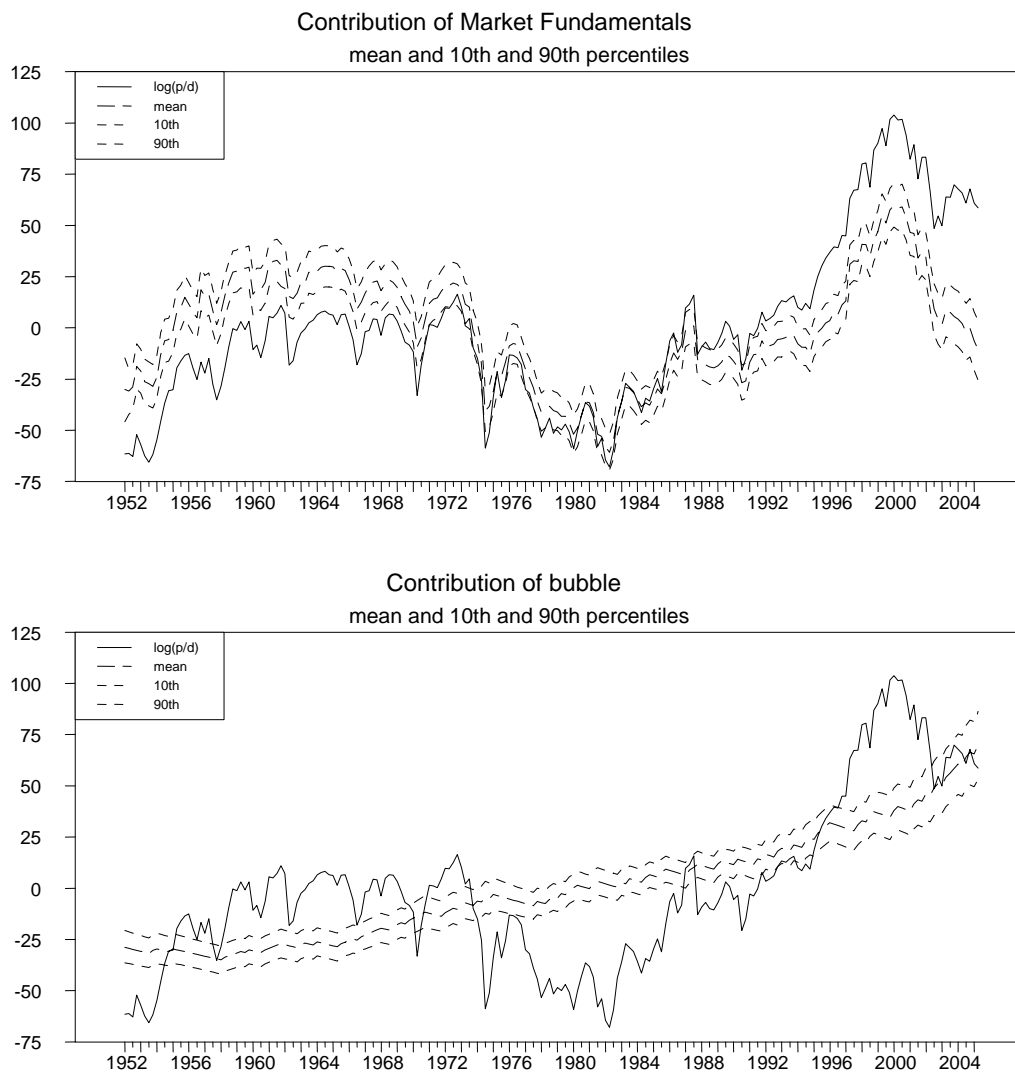


Figure 7. Posterior Distribution of Contribution of Market Fundamentals and Bubble to Log Price-Dividend Ratio: Model with a permanent dividend growth factor (“uninformed” prior on variance of period-to-period shocks to bubble).
Sample means removed.

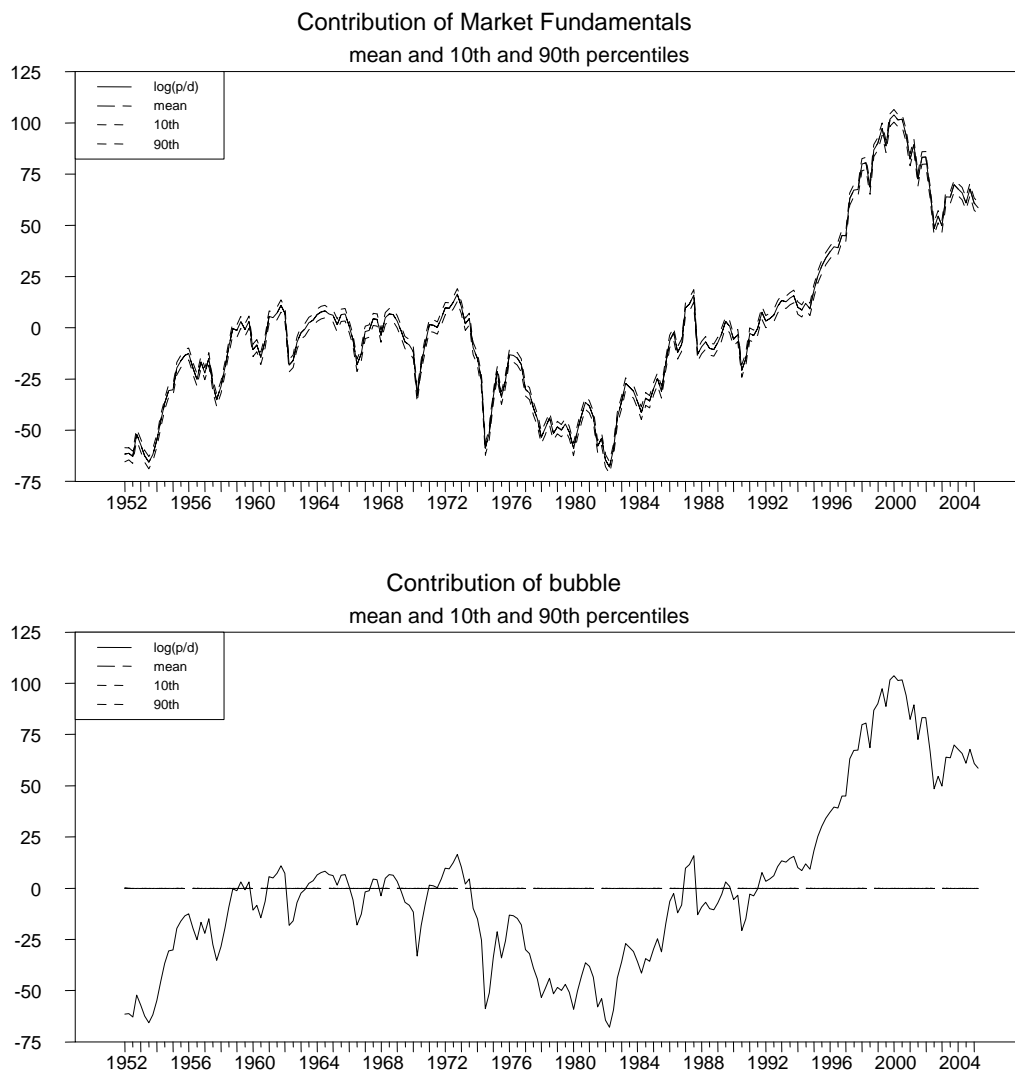


Figure 8. Posterior Distribution of Contribution of Market Fundamentals and Bubble to Log Price-Dividend Ratio: Model with permanent component for required returns (“uninformed” prior on variance of period-to-period shocks to bubble).
Sample means removed.

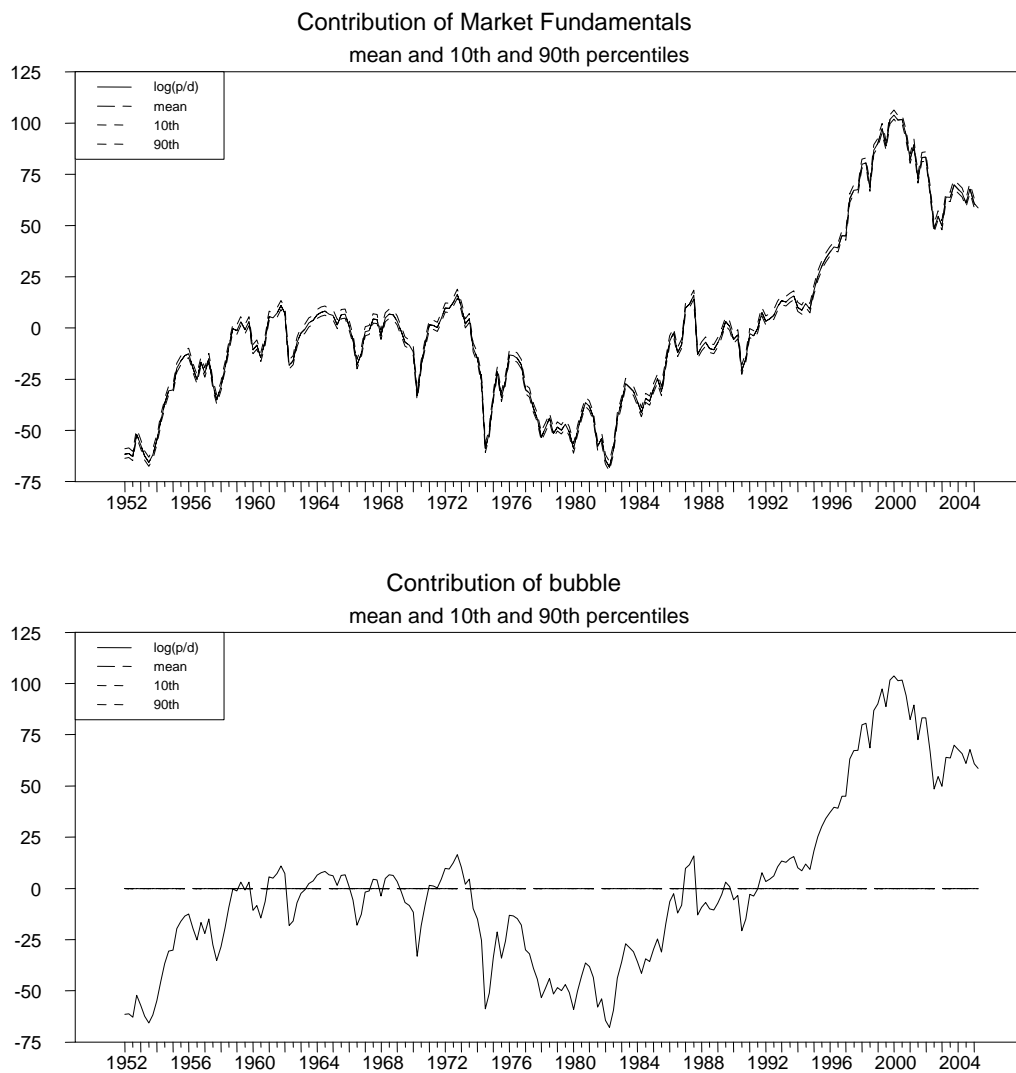
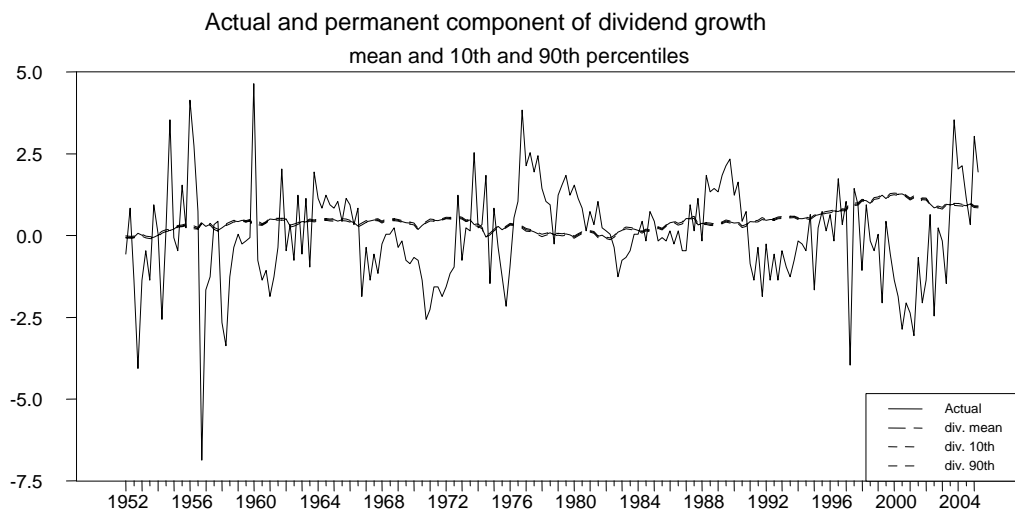


Figure 9. Posterior Distributions of Permanent Market Fundamental Factors

Panel A. Model with permanent component of dividend growth (“uninformed” prior on variance of period-to-period shocks to bubble).
Sample means removed.



Panel B. Model with permanent component of required returns (“uninformed” prior on variance of period-to-period shocks to bubble).
Sample means removed.

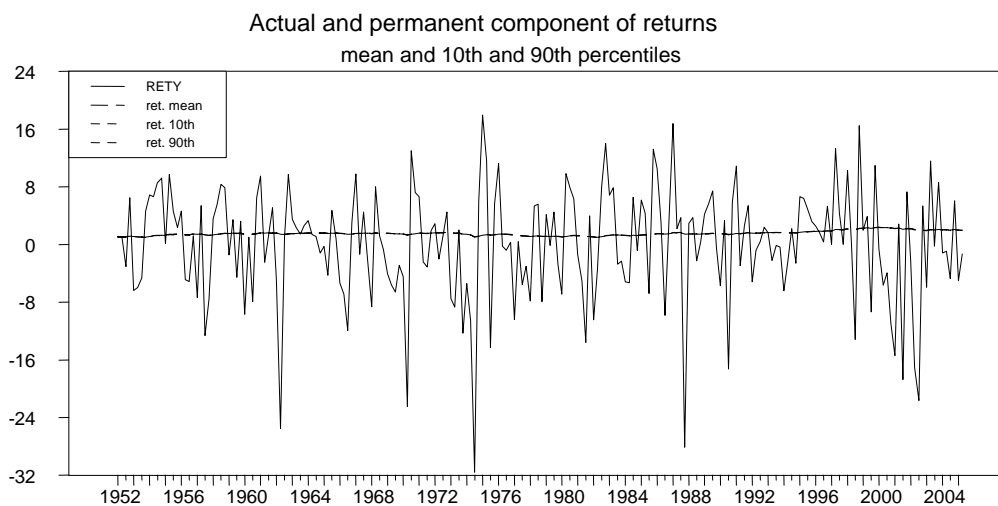


Figure 10. Posterior Distribution of Contribution of Market Fundamentals and Bubble to Log Price-Dividend Ratio: Model includes two permanent factors (“uninformed” prior on variance of period-to-period shocks to bubble).
Sample means removed.

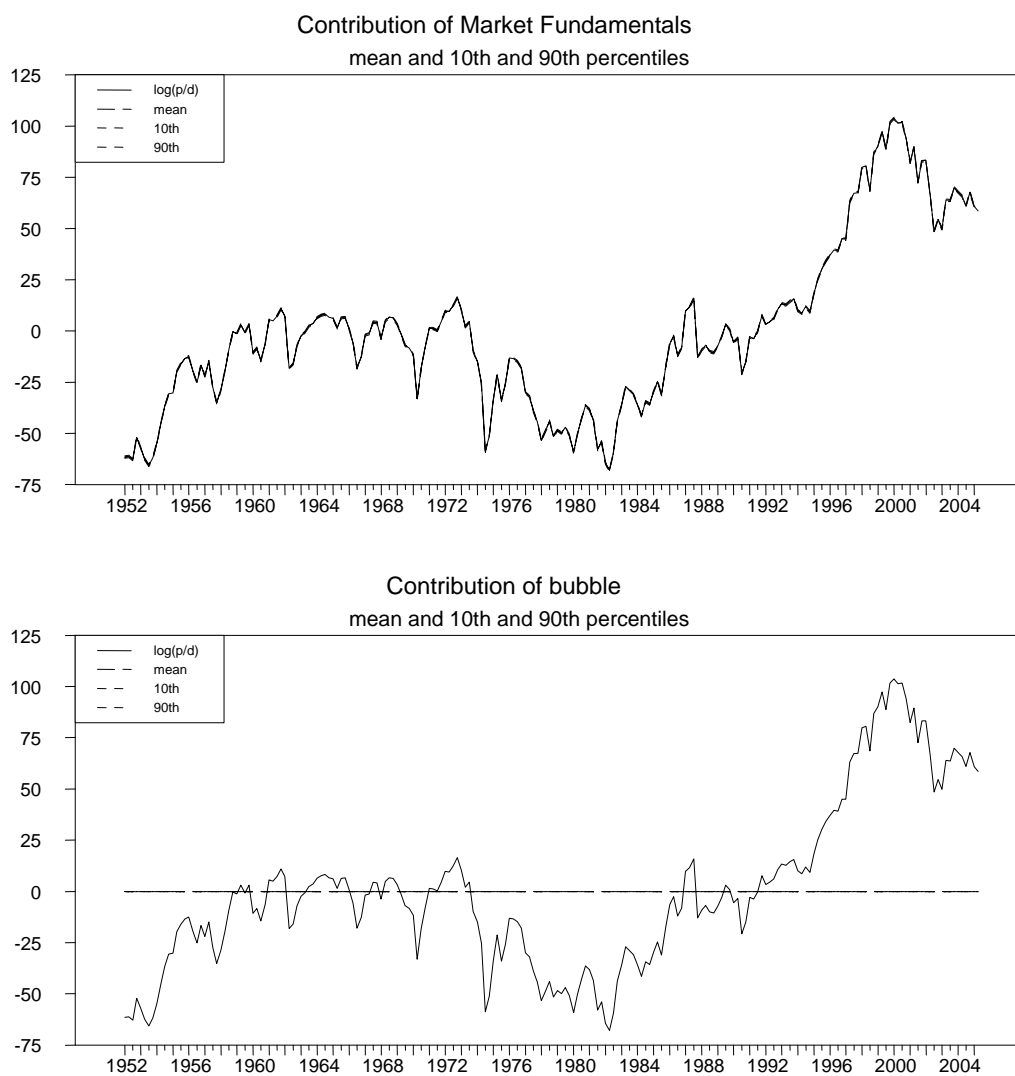


Figure 11. Posterior Distribution of permanent factors for dividend growth and required returns: Model with two permanent factors (“uninformed” prior on variance of period-to-period shocks to bubble). Sample means removed.

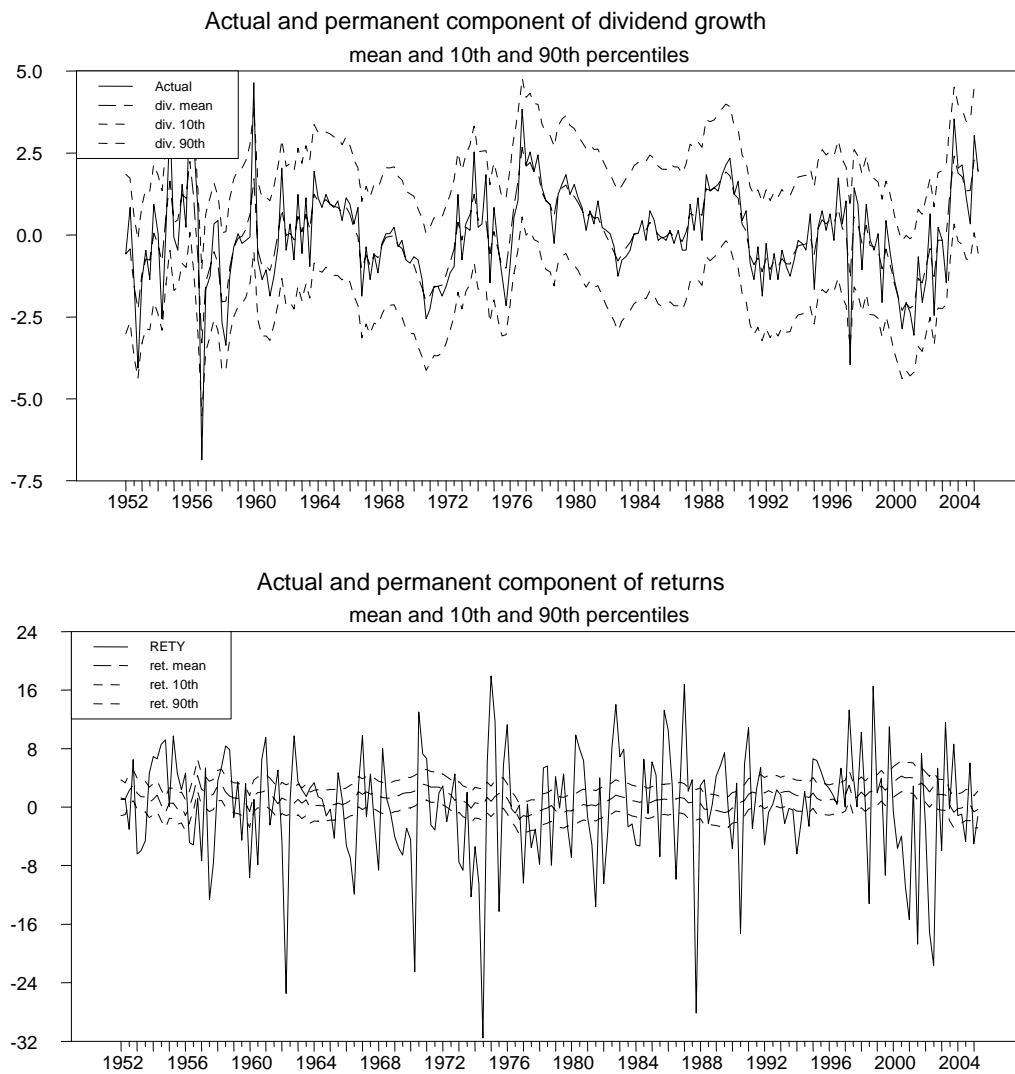


Figure 12. Hypothetical path of log price dividend when a bubble is present in the Gordon model and when a bubble is present in the Campbell-Shiller approximation.

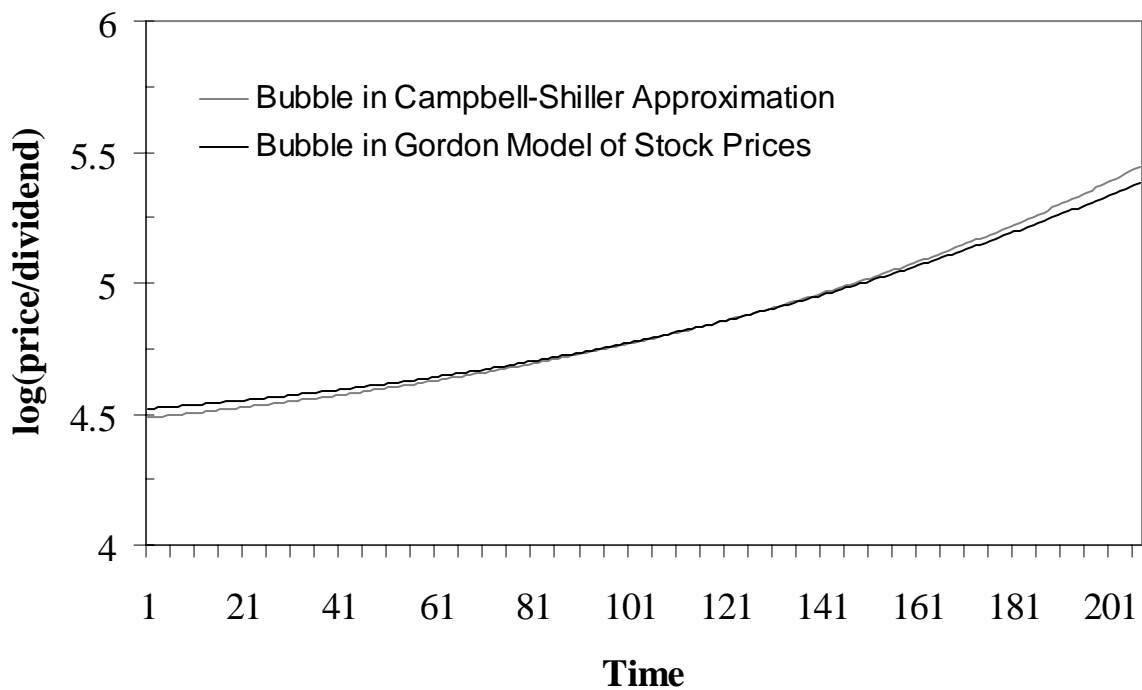


Figure 13. Posterior Distribution of Contribution of Market Fundamentals and Bubble to Log Price-Dividend Ratio: Model includes two “persistent” factors (“uninformed” prior on variance of period-to-period shocks to bubble).
Sample means removed.

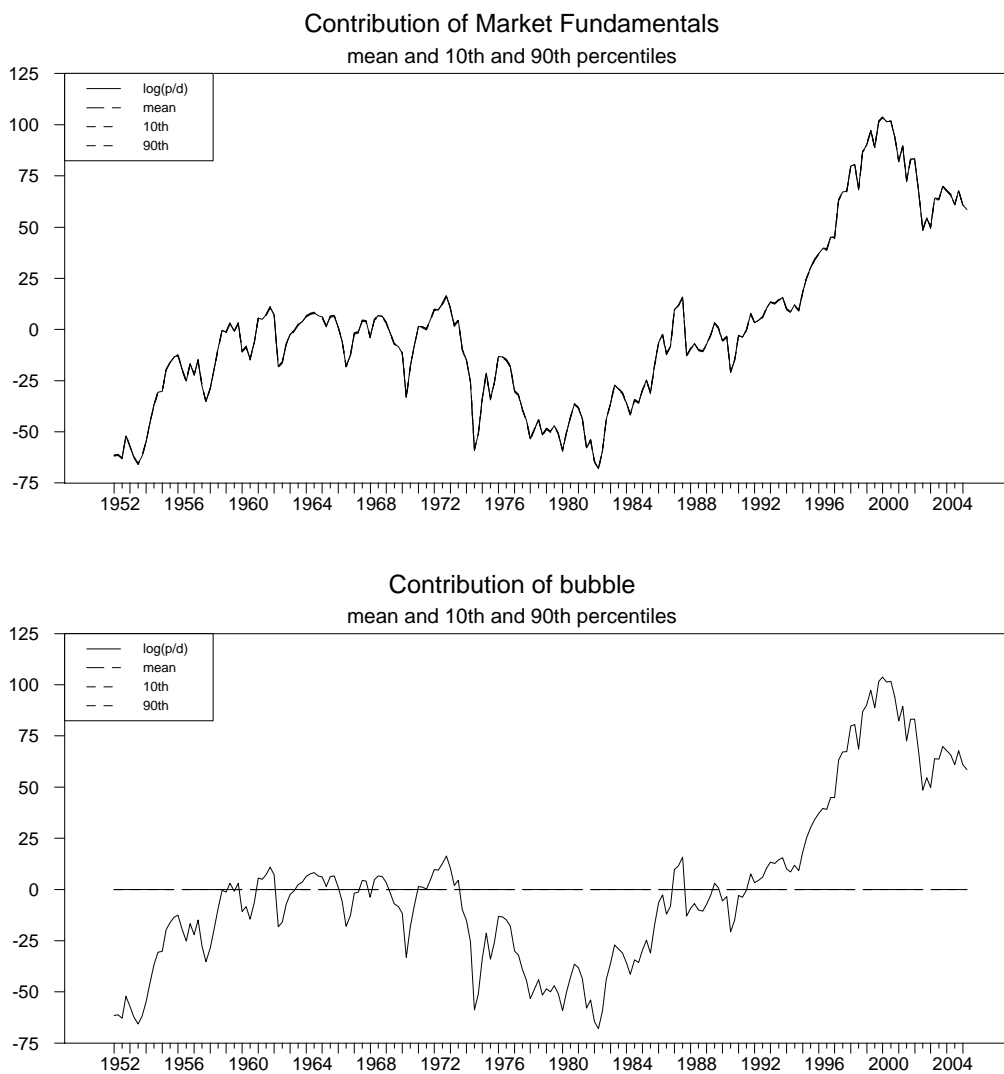


Figure 14. Posterior Distribution of “persistent” factors for dividend growth and required returns: Model with two “persistent” factors (“uninformed” prior on variance of period-to-period shocks to bubble). Sample means removed.

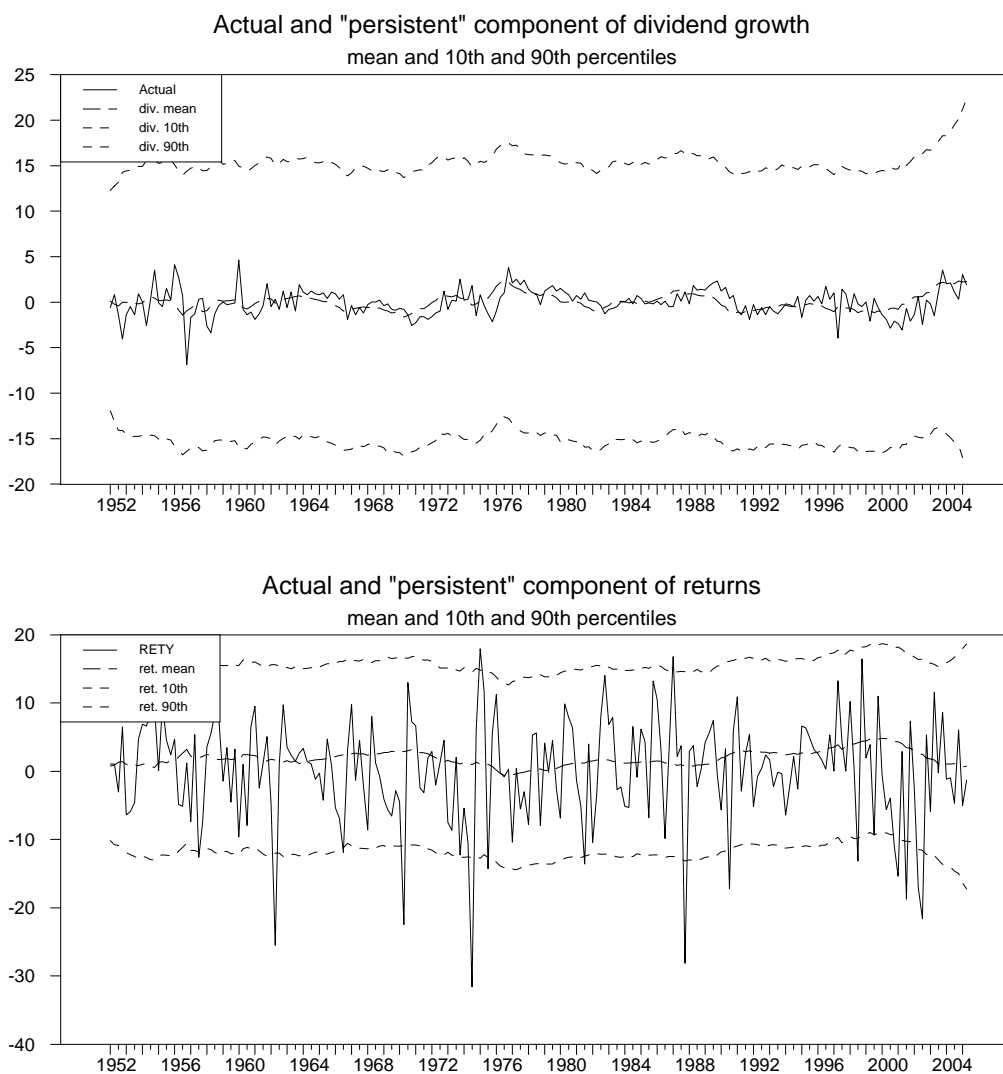


Figure 15. Posterior Distribution of Contribution of Market Fundamentals and Bubble to the Log Price-Earnings Ratio: Model includes two permanent factors (“uninformed” prior on variance of period-to-period shocks to bubble).
Sample means removed.

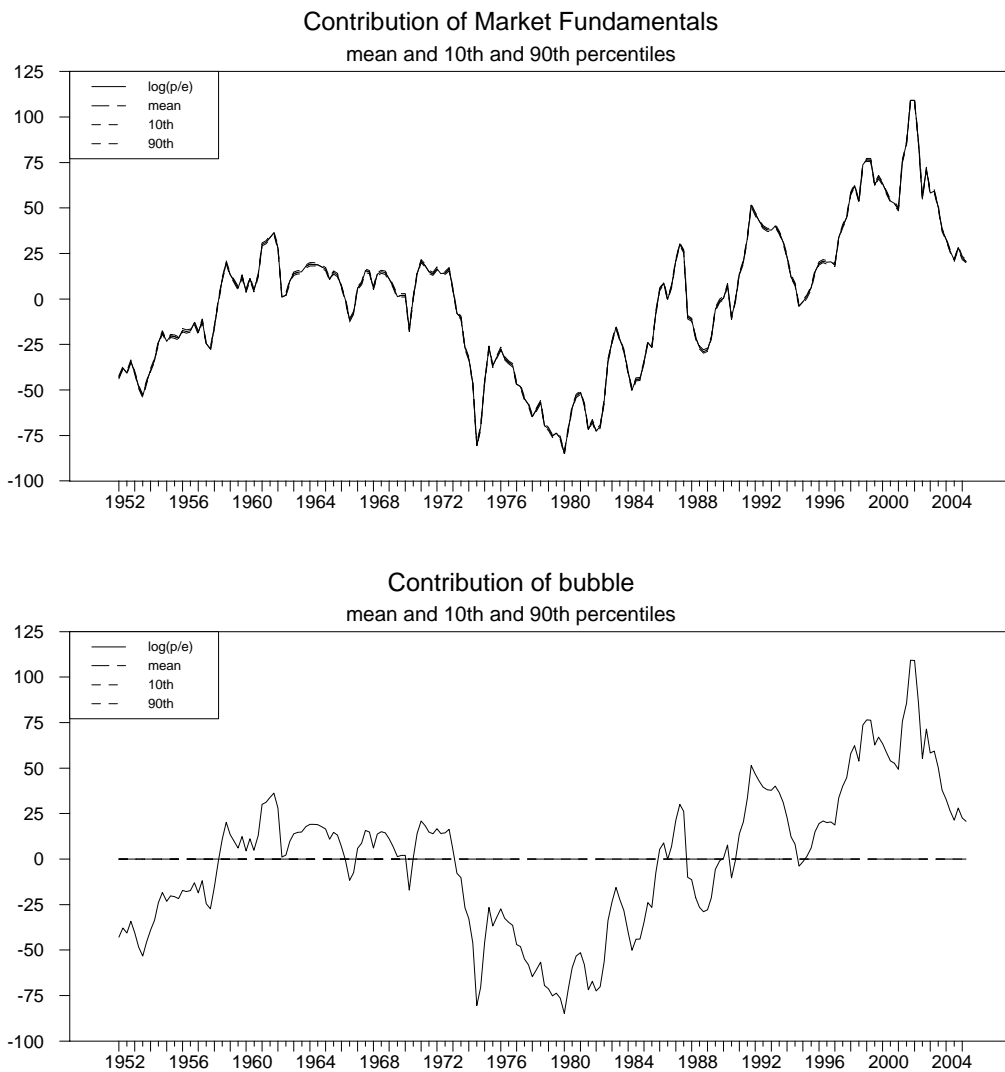


Figure 16. Posterior Distribution of Contribution of Market Fundamentals and Bubble to Log Price-Earnings Ratio: Model with stationary earnings growth, required returns, and “strong” prior beliefs that the variance of period-to-period shocks to bubble (σ_b^2) is “small”.

Sample means removed.

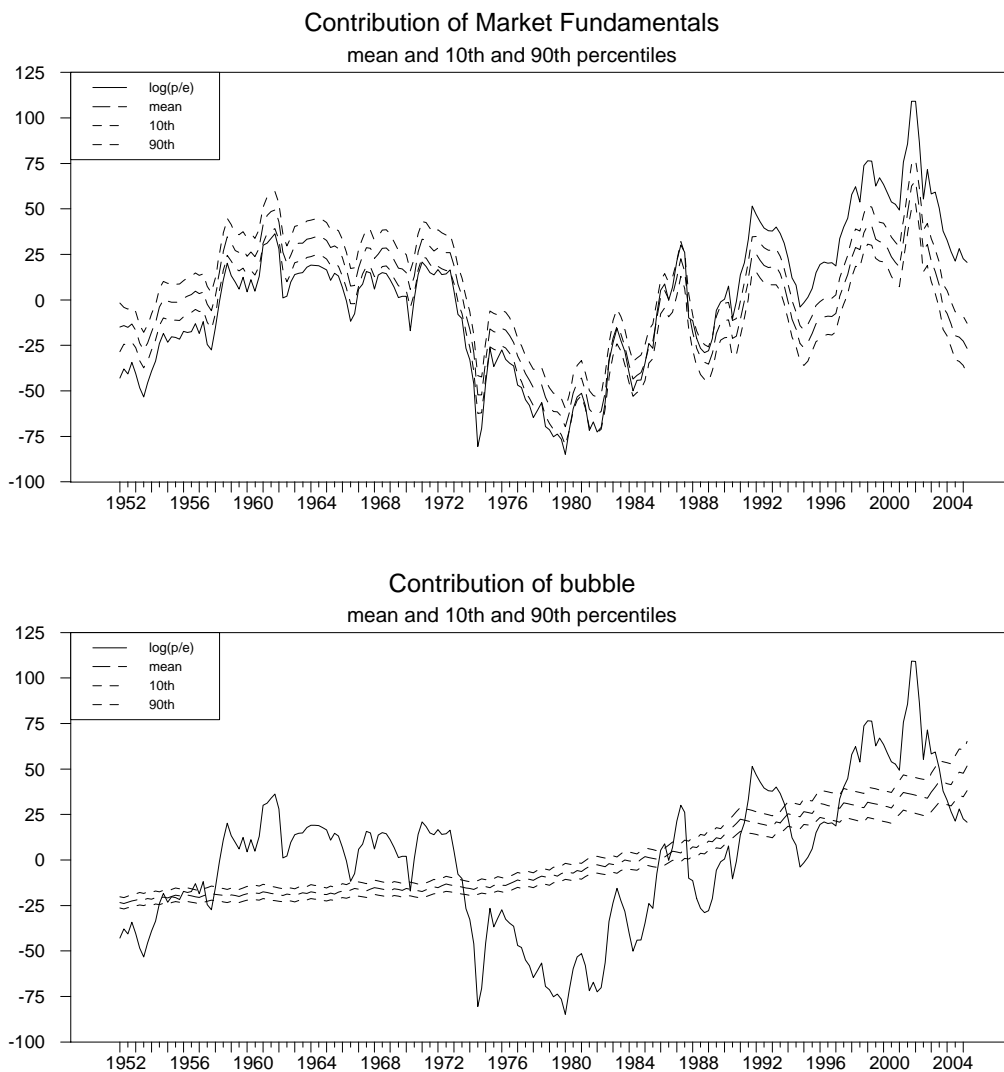


Figure 17. Posterior Distribution of Contribution of Market Fundamentals and Bubble to Log Price-Dividend Ratio for NASDAQ: Model includes two permanent factors (“uninformed” prior on variance of period-to-period shocks to bubble).
Sample means removed.

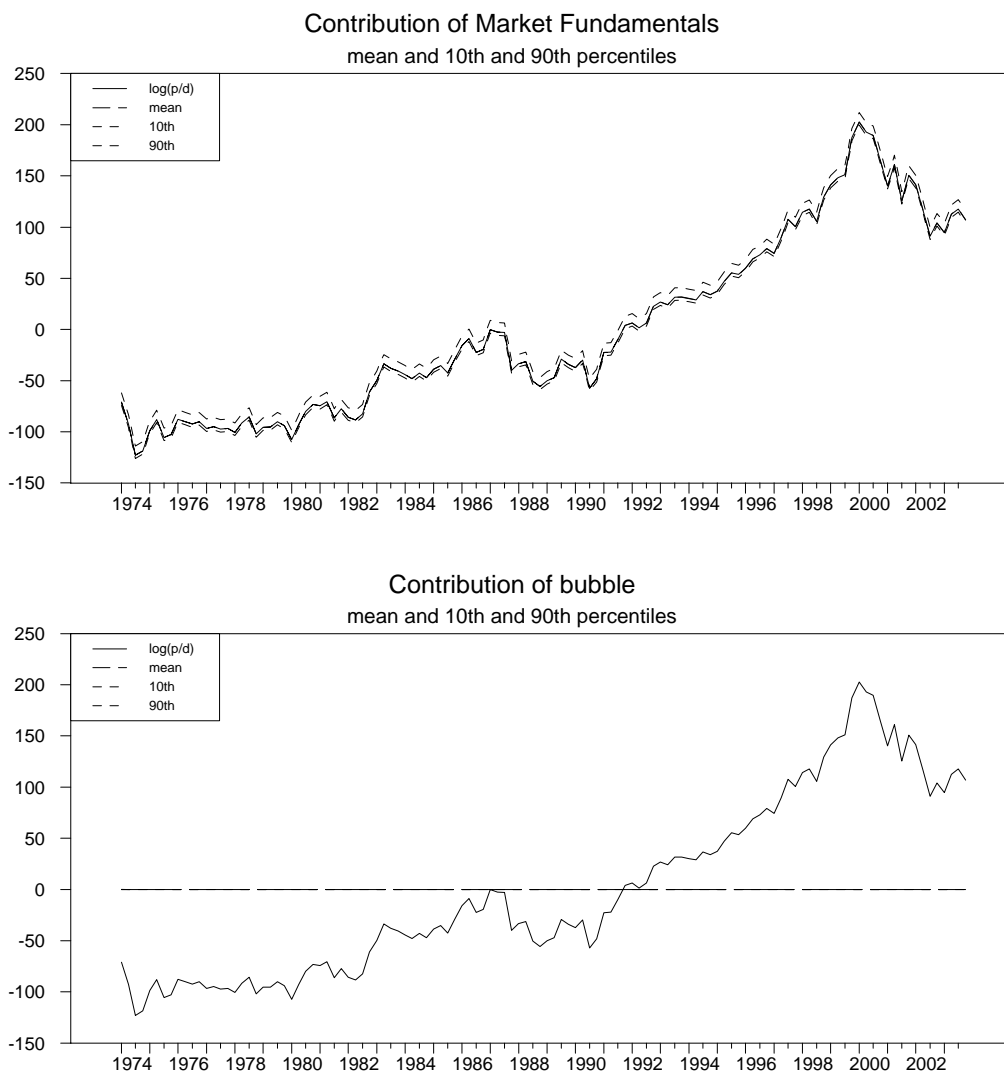


Figure 18. Posterior Distribution of permanent factors for dividend growth and required returns using NASDAQ index: Model with two permanent factors (“uninformed” prior on variance of period-to-period shocks to bubble).
Sample means removed.

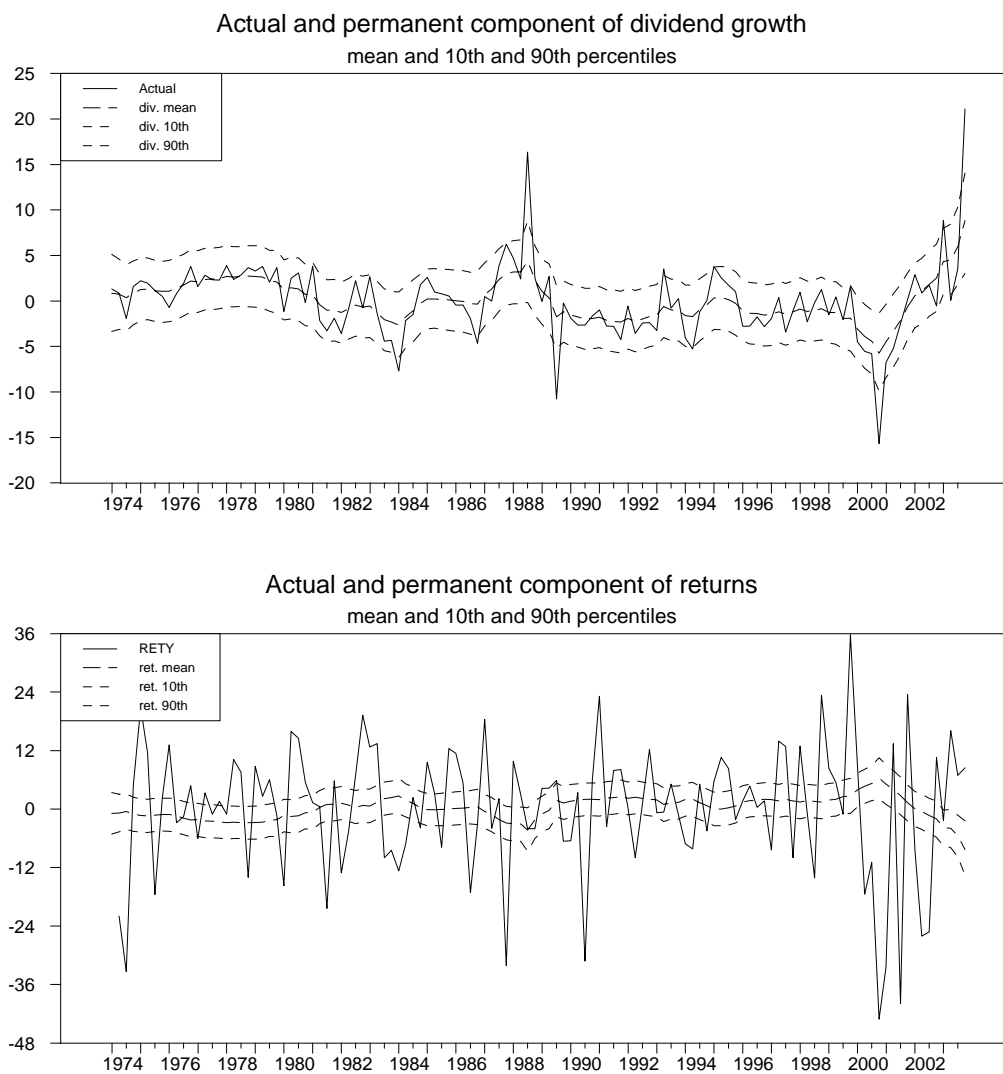


Figure 19. Posterior Distribution of Contribution of Market Fundamentals and Bubble to Log Price-Dividend Ratio for NASDAQ: Model with stationary dividend growth, required returns, and “strong” prior beliefs that the variance of period-to-period shocks to bubble (σ_b^2) is “small”.

Sample means removed.

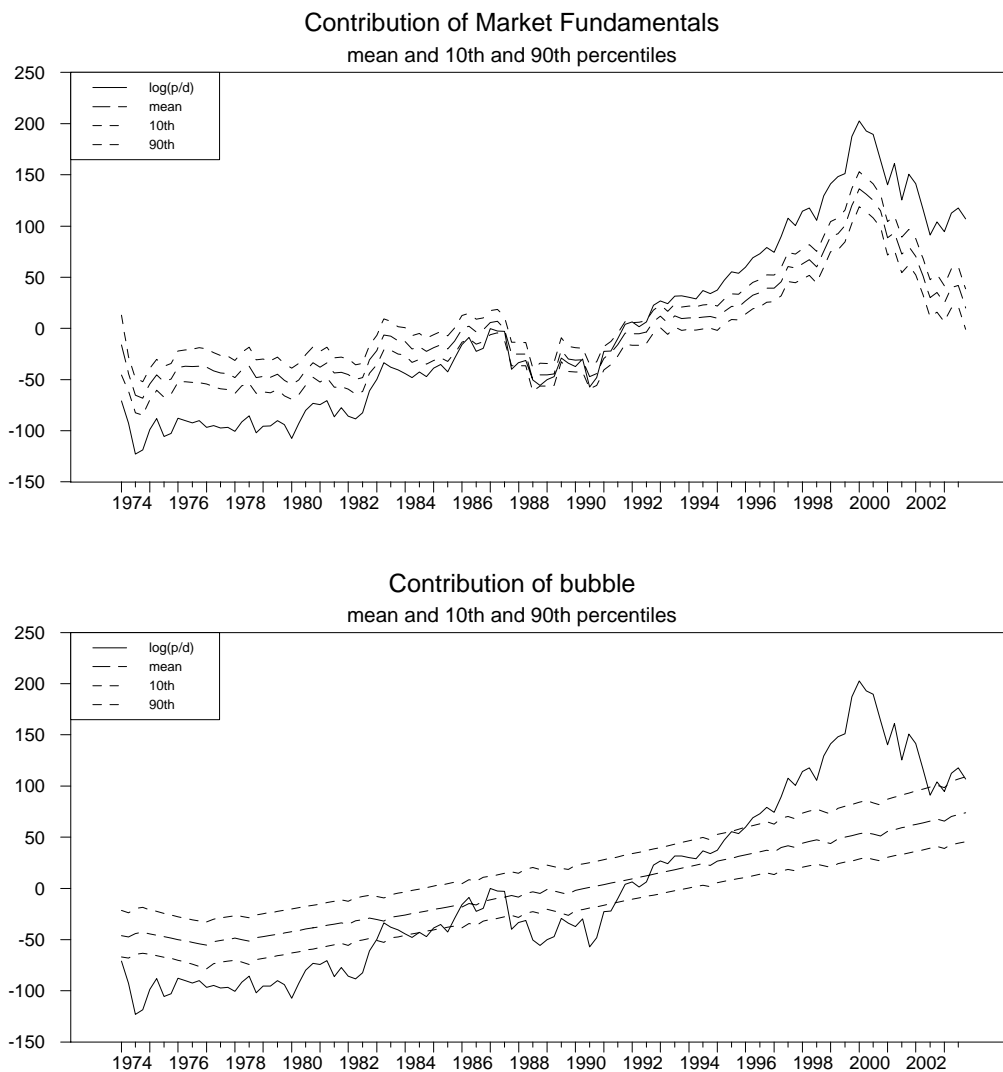


Figure 20. Probability of being in explosive bubble regime for NASDAQ

Model with stationary dividend growth, required returns, and “strong” prior beliefs that the variance of period-to-period shocks to bubble (σ_b^2) is “small”.

