A Cointegrated Structural VAR Model of the Canadian Economy

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[1]We would like to thank participants at the 2001 WEA International meetings in San Francisco for valuable comments. All remaining errors are our own.

Abstract

This paper implements a cointegrated structural VAR model of the Canadian economy using quarterly data over the period 1964-1994. The dynamic properties of the estimated model are compared to the predictions of a simple textbook macro model. Four long-run equilibrium relationships; i) consumption-income, ii) consumption-wealth, iii) money demand and iv) the Fisher equation are tested. Our empirical results are generally consistent with the predictions of the textbook model's long-run implications, although level shifts are observed in the consumption/income and the wealth/income ratios. Similarly we find that there was an increase in the ex-post real interest rate, implying a level shift in the Fisher relation, following the Bank of Canada's policy change toward a stable price level target.

Keywords: Cointegration, Structural VAR, Business Cycle.

JEL Classifications: C22,E3,E4.

1 Introduction

Despite their stylized nature and simplicity, basic textbook macroeconomic models often form the basis of the responses of professional economists when asked to describe the effects of different policies or shocks (e.g. increases in productivity or decreases in money growth) on such variables as real output, interest rates, inflation, consumption and wealth. These simple models often provide reasonably accurate descriptions of how the U.S. economy works and there exist a number of empirical studies analyzing such macroeconomic models for the U.S. economy. For example, Sims (1980) employs reduced form vector autoregressions (VARs) to investigate the behavior of some important macro variables within a dynamic framework. His results supported the monetarists view of how the economy functions. In response to criticisms of reduced form VARs with regards to econometric identification (see Leamer, 1985 and Cooley and LeRoy, 1985), Blanchard (1989) imposed restrictions on the contemporaneous correlations among the endogenous variables in an effort to identify the structural innovations from the reduced form model of the U.S. economy. Blanchard and Quah (1989) use long-run neutrality restrictions to identify aggregate demand and aggregate supply shocks. Gali (1992) achieves identification by imposing both long-run (neutrality) and short-run (contemporaneous) identifying restrictions on a small VAR model based on the Keynesian IS-LM model. King, Plosser, Stock and Watson (1991) show how the stochastic trend components of "permanent" innovations can be identified in a cointegrated system. Crowder, Hoffman and Rasche (1999) expand on King et. al. (1991) by imposing a complete structural identification of components that leave both permanent and transitory effects on the variables of interest. All of the above studies generate results that are generally consistent with predictions of testbook macroeconomic models. [1]

This paper follows the framework used in King, et al. (1991) and Crowder, et al. (1999) and extends the model by incorporating consumption and equity wealth into a small macro model for Canada. The empirical model yields four equilibrium relationships, a consumption function, a money demand relation, a Fisher relation and a long-run link between equity wealth and income. Theory suggests that each of the above relations should have a stable long-run equilibrium.

The results of this paper show that, while the long-run linkages between these relationships were maintained over the entire sample period, after 1982 permanent level shifts occurred in the consumption-to-income ratio, the equity wealth-to-income ratio and ex-post real interest rates. In particular, we find a downward shift in the consumption-to-income ratio from an average of 0.907 during the pre-1982 period, to an average of 0.866 during the post-1982 period. Using recently developed measures of equity wealth for Canada, we also find a downward shift in the total consumption-to-wealth ratio from an average of 1.248 during the pre-1982 period, to an average of 0.741 during the post-1982 period. This is consistent with the literature indicating an increase in private saving and equity wealth during this period as well as a slowdown in the growth of consumption. Finally, ex-post real interest rates increased from an average of 0.34% during the pre-1982 to an average of 6.28% during the post-1982. We also find evidence of the Darby effect (effect of taxes on the Fisher coefficient) in the Fisher relation, such that we cannot reject a Fisher coefficient of 1.6. We identify our break points both statistically and historically by noting that the Canadian economy suffered a severe recession from 1980 to 1982. In addition, the Canadian central bank pursued a tight monetary policy and expansionary fiscal policy during the post-1980 period. In the post-1980 period Canada also increased incentives to save through changes in the tax laws. The above results are consistent with the literature suggesting that the Central Bank of Canada maintained a high level of real interest rates to keep inflation down and encourage private saving.

The remainder of this paper is organized as follows: Section 2 provides the theoretical foundation that underlies our subsequent empirical results as well as a short literature review. Section 3 describes the econometric methodology employed. Section 4 describes the data and the empirical results. Section 5 provides a summary and conclusion.

2 The Empirical Model

We chose six variables to include in our empirical model of the Canadian economy. These include log real per-capita disposable personal income, log real per-capita consumption expenditures, log real per-capita M1 money balances, the three-month Canadian treasury bill interest rate, the GDP deflator inflation rate and log real per-capita equity wealth. These variables were chosen not only for their importance in the functioning of the macroeconomy, but also for the specific questions of interest in our analysis. Real income, real money and nominal interest rate are standard in almost all small scale models of the macroeconmy. We add real consumption and real equity wealth to investigate the welth effect on consumption and the other macro variables. We include inflation to capture the indirect effects of nominal policy shocks, especially monetary policy. These variables were also chosen because of the theoretical equilibria shared among them.

2.1 Four Long-run Equilibrium Relations

The following long-run equilibrium relationships can easily be derived from standard textbook models of the macroeconomy:

1. Consumption-Income: $\mathrm{c}_t = \theta_1 \mathrm{y}_t$

2. Consumption-Wealth: $\mathrm{c}_t = \theta_2 \mathrm{w}_t$

Hypothesized Values: $\beta_1 = \theta_2 = 1$

3. Money Demand Relation: $\mathrm{m}_t = \gamma_1 \mathrm{y}_t - \gamma_2 i_t$

   Hypothesized Values: $\gamma_1 = 1$

4. Tax-Adjusted Fisher Relation: $r_t = (1 - \tau) i_t - \pi_{t+1}$

Our system consists of six variables; y, π, i, c, w, and m, with four hypothesized long-run equilibrium conditions. In the empirical section of this study, we will demonstrate that the Canadian data comprising these six variables are consistent with stochastic trend behavior implying that the hypothesized equilibria can be interpreted econometrically as four cointegrating relations among the six variables. This implies that all but two of the innovations impinging on the data have only transitory effects. This yields a convenient interpretation for the two shocks that leave permanent imprints on the data, ie. a permanent real innovation and a permanent nominal innovation. Identification of the two permanent shocks relies on a long-run neutrality restriction. The theoretical predictions of our model with respect to the behavior of the endogenous variables to each permanent innovation can be summarized as follows:

Permanent real shocks (e.g. productivity shocks or aggregate supply shocks) will have permanent effects on all of the variables. A positive innovation to productivity should permanently increase real income, real money balances, real consumption and real equity wealth. The effect on nominal interest rates and inflation should be to reduce both in the long run but consistent with a temporary increase in real interest rates.

Permanent nominal shocks will have at most only temporary effects on real income, real consumption, real equity wealth and real interest rates. Such shocks should leave a permanent imprint on nominal interest rates and inflation and real money balances.

This leaves four transitory or temporary innovations. Identification of the four transitory innovation is achieved by a set of contemporaneous restrictions on the effects of the four innovations.

2.2 The Consumption-Income Relation

The aggregate consumption function has been a key component of macroeconomic models, dating back to Keynes. The Keynesian absolute income hypothesis proved to be inadequate in explaining actual consumption behavior. This led economists to develop new theories of consumption, most notably, the Permanent Income Hypothesis (PIH) of Friedman (1957) and the life-cycle hypothesis (LCH) of Ando and Modigliani (1963). These theories are based on rational utility maximizing behavior in an intertemporal environment.

Wirjanto (1995) investigates the intertemporal consumption decisions of individuals in a general PIH model in which a fraction of the consumers are liquidity constrained. The liquidity-constrained individuals are assumed to be able to borrow against their future labor income, with the rate at which they borrow being an increasing function of the loan size. This assumption implies that the growth rate of consumption is related to the growth rate of income and to the level of real interest rate. If consumption and income are integrated of order one or I(1), and the real interest rate is stationary or I(0), then this would imply a cointegrating vector of [1,-1] between consumption and income.

2.3 Consumption-Equity Wealth Relation

The PIH implies that in a stylized neo-classical environment, consumption is a constant proportion of lifetime wealth. In a stochastic environment this relationship would find a steady state equilibrium with cointegrating relationship between consumption and wealth of [1,-1], assuming both variables are I(1). Presumably this would mirror the relationship between consumption and (permanent) income. Since this property of consumption is already being captured in the consumption-income relation, we examine a more narrow definition of wealth that is of independent interest, i.e. equity wealth.

A burgeoning literature has arisen examining the so-called equity wealth effect on consumption spending, e.g. Ludvigson and Steindel (1999). The evidence to date is almost exclusively from the U.S. and is mixed with a general conclusion that changes in equity wealth have a small effect on consumption spending. These results, however, have been obtained without carefully treating the stochastic trend properties of the underlying data and are suspect. We attempt to learn something of the consumption-equity wealth relation for Canada using a wealth measure developed by Maklem (1997). While we impose no particular long-run equilibrium on the variables, the PIH implies a [1, -1] relationship between total wealth and consumption. We believe that there is no reason that equity wealth should accumulate at a rate different from total wealth in a steady state equilibrium and thus test whether the theoretical relationship between consumption and total wealth also holds for consumption and equity wealth. Once we have adequately treated the stochastic trend behavior of the variables, we can examine the short-run dynamic relationship between equity wealth and consumption in a fully specified structural VAR.

2.4 Money Demand Relation

The early literature examining the Canadian money demand function focused primarily on a narrow monetary aggregate and found relatively weak evidence of a stable relation, e.g. Clark (1973), Foot (1977) and Boothe and Poloz (1988). The more recent literature, employing a broader definition of money and using cointegration methodology (suited for non-stationary variables) found stronger evidence of a stable money demand function in Canada, e.g. Hoffman, Rasche and Tieslau (1995) and Haug and Lucas (1996). The equilibrium money demand relationship is important as it ties together real income and nominal interest rates in the long run. Important dynamics may be missed if this relationship is left unmodelled.

2.5 The Fisher Hypothesis

With respect to the Fisher equation, there has been little evidence of the Fisher effect in other countries other than the U.S. Mishkin (1984) examines real interest rates in Canada and six other OECD countries and imposes a Fisher effect, i.e. the response of nominal rates to changes in (expected) inflation, equal to unity. His analysis thus, cannot be interpreted as a direct test of the Fisher relation. MacDonald and Murphy (1989) provide direct tests of the Fisher relation for four countries, including Canada. While they find cointegration between the Canadian nominal interest rate and inflation, the standard error on the Fisher coefficient is too large to make any conclusions with respect to the long-run Fisher relation. Dutt and Ghosh (1995), also employing cointegration methods, investigate the Fisher effect for Canada. They separate their sample into two periods; one corresponding to the post-Bretton Woods period of flexible exchange rates, and the second the fixed exchange rate Bretton Woods period. They find no evidence supporting the Fisher hypothesis in either period. Crowder (1997), extending his earlier work on the U.S., utilizes more advanced time series methods and finds evidence of a strong long-run relation between the Canadian nominal 3-month commercial paper interest rate and inflation (computed using GDP price deflator). Daly and Jung (1997) estimate Canadian marginal tax rates in a range that suggests the tax-adjusted Fisher effect in Canada should lie in the between 1.5-1.9. Crowder estimates the Fisher effect in Canada to be between 1.51 and 1.81 depending on how structural instability in the relationship is treated.

3 Empirical Methodology

3.1 Univariate Analysis

Two of the most commonly used unit root tests in the literature are the augmented Dickey-Fuller test (ADF τ-test) of Said and Dickey (1984) and Phillips-Perron test (PP Z-test) developed in Phillips and Perron (1988). It is well known that the ADF and PP tests have low power against local stationary alternatives. Elliot, Rothenberg and Stock (1996) (ERS DF^{GLS}) develop a feasible point optimal test that relies on local GLS detrending to increase the power of the unit root tests.

A second serious problem associated with unit root testsing is that the above named tests all suffer from serious size distortions when the data generating process (DGP) has negative moving average terms. Schwert (1987, 1989), Phillips and Perron (1988), Pantula (1991), Ng and Perron (1995, 2000) and Perron and Ng (1996) demonstrate that the empirical size of conventional ADF and PP tests approach unity as the sum of the MA parameters in a univariate process approach negative one. Perron and Ng (1996) extend the work of Elliot, Rothenberg, and Stock (1996) by developing modified versions of the PP tests that have much better size properties than the conventional PP tests but also retain the power of the (ERS DF^{GLS}). These unit root tests are based on the local GLS detrending method and in addition use an autoregressive spectral density estimator of the long-run variance. The two tests are labeled the MZ_ρ and the MZ_τ test. Ng and Perron (2000) suggest that these two tests have similar power to the DF^{GLS} test of Elliot, Rothenberg and Stock (1996) but also have superior size properties in the presence of MA disturbances. The decrease in size and increase in power are enhanced when one chooses the lag length based on the modified information criteria (MIC) developed in Ng and Perron (2000).

The tests proposed by Ng and Perron are motivated by the DGP in (1),

$$y_t = d_t + u_t, \qquad u_t = \rho u_{t-1} + v_t \tag{1}$$

where $v_t = \varphi(L)e_t = \sum_{j=0}^{\infty} \varphi_j e_{t-j}, d_t = \zeta' z_t = \sum_{i=0}^{p} \zeta_i t^i$ for $p = 0, 1$. ERS suggest using a GLS detrending method to improve the power of unit root tests. For any series $\{x_t\}_{t=0}^{T}$ define $(x_0^{\overline{\alpha}}, x_t^{\overline{\alpha}})$ $\equiv (x_0, (1 - \overline{\alpha}L)x_t)$ for some chosen $\overline{\alpha} = 1 + \overline{c}/T$. The GLS detrended series is defined as,

$$\widetilde{y}_t \equiv y_t - \widehat{\zeta}' z_t \tag{2}$$

where $\widehat{\zeta}$ minimizes $S(\overline{\alpha}, \zeta) = (y^{\overline{\alpha}} - \zeta' z_t^{\overline{\alpha}})'(y^{\overline{\alpha}} - \zeta' z_t^{\overline{\alpha}})$. ERS suggest choosing $\overline{c} = -7.0$ for $p = 0$ and $\overline{c} = -13.5$ for $p = 1$. The test recommended by ERS is the DF^{GLS} statistic given in equation (3).

$$\Delta \widetilde{y}_t = \rho \widetilde{y}_{t-1} + \sum_{j=1}^{k} \gamma_j \widetilde{y}_{t-j} + e_{tk} \tag{3}$$

Ng and Perron recommend two tests that have similar power to the DF^{GLS} but that also have superior size properties in the presence of MA errors. These tests are MZ_ρ, MZ_t, and MSB, collectively referred to as the M tests. These are defined as,

$$MZ_\rho = (T^{-2}\widetilde{y}_t^2 - s_{AR}^2)(2T^{-2}\sum_{t=1}^{T} \widetilde{y}_{t-1}^2)^{-1} \tag{4}$$

$$MSB = \left[\frac{T^{-2}\sum_{t=1}^{T} \widetilde{y}_{t-1}^2}{s_{AR}^2}\right]^{\frac{1}{2}} \tag{5}$$

and $MZ_t = MZ_\rho \times MSB$. All three tests are based on s_{AR}^2, an autoregressive estimate of the spectral density at frequency zero of v_t. This estimate is calculated as,

$$s_{AR}^2 = \frac{\widehat{\sigma}_k^2}{[1-\gamma(1)]^2} \tag{6}$$

where $\gamma(1) = \sum_{i=1}^{k} \gamma_i$ and $\widehat{\sigma}_k^2 = (T-k)^{-1}\sum_{t=k+1}^{T} \widehat{e}_{tk}^2$ and γ_i and $\{\widehat{e}_{tk}\}$ are taken from estimation of (3) using OLS. The only piece left is to specify a lag truncation parameter k. Ng and Perron suggest using a modified information criteria (MIC) as in (7),

$$MIC(k) = \ln(\widehat{\sigma}_k^2) + \frac{C_T(\tau_T(k)+k)}{T-k_{\max}} \tag{7}$$

where $\tau_T(k) = (\widehat{\sigma}_k^2)^{-1}\widehat{\rho}\sum_{t=k_{\max}+1}^{T} \widetilde{y}_{t-1}^2$ and $k_{\max}$ is the largest lag truncation considered. Ng and Perron show that if $C_T = \ln(T-k_{\max})$ then this reduces to their $MBIC$ or modified Bayesian information criteria.

3.2 Multivariate Analysis

As discussed above, there are four equilibrium relationships to test, implying statistical cointegration among the integrated variables. These four equilibria are; i) log real per-capita consumption (c) and log real per-capita disposable income (y), implied by a balanced growth restriction, which is hypothesized to be equal to [1,-1], ii) (log) real per-capita consumption and (log) real per-capita equity wealth (w), implied by the permanent income hypothesis, which is hypothesized to be equal to [1,-1], iii) a money demand function with unitary income elasticity and a negative interest elasticity that is less than one in absolute value and iv) the 3-month T-bill interest (i) rate and inflation (π) that is in the range of [1,-1.5] to [1,-1.9], implied by the Fisher hypothesis. These four hypotheses are tested using the maximum likelihood procedure proposed by Johansen (1988, 1991).

This procedure is based upon the vector error correction model (VECM) in equation (8),

$$\Delta X_t = \mu + \alpha\beta' X_{t-1} + \sum_{j=1}^{k-1} \Gamma_j \Delta X_{t-j} + \varepsilon_t \tag{8}$$

where X_t is a p x 1 vector of at most I(1) series,[2] β is a p x r matrix whose r columns represent the cointegrating vectors among the variables in X_t, and α is a p x r matrix whose p rows represent the error correction coefficients. The Johansen test for cointegration, called the trace test, tests the rank of the p x p product matrix $\alpha\beta'$ such that reduced rank, rank less than p, implies cointegration.

Maximum likelihood estimation of (8) can be carried out by applying reduced rank regression. Johansen (1988, 1991) suggests first concentrating out the short-run dynamics by regressing ΔX_t and X_{t-1} on ΔX_{t-1}, ΔX_{t-2}, ..., ΔX_{t-k+1} and μ and saving the residuals as R_{0t} and R_{1t}, respectively. Calculate the product moment matrices $S_{ij} = T^{-1} \sum_{t=1}^{T} R_{it} R_{jt}^{'}$ and solve the eigenvalue problem $\left| \lambda S_{11} - S_{10} S_{00}^{-1} S_{01} \right| = 0$. The likelihood ratio statistic testing the null hypothesis of at least r cointegrating relationships in X_t is given by $-T \sum_{i=r+1}^{p} \ln(1 - \widehat{\lambda}_i)$ and is called the trace statistic by Johansen (1988, 1991). The distribution of this statistic is non-standard and depends upon nuisance parameters. Critical values have been tabulated by MacKinnon et al. (1996) using response surface regressions.

The trace test takes the null hypothesis of at least r cointegrating vectors in X_t versus the alternative of $r + 1$ cointegrating vectors. The term μ captures the deterministic drift in the series that is eliminated by the cointegration vector(s). It is possible that the series have no drift, but the cointegration vector has a non- zero mean, in which case $\mu = 0$ and a column of ones is appended to X_t. If the cointegration vector has a zero mean and the series have no drift, then $\mu = 0$ and X_t will not contain any deterministic components. These distinctions are important, since the asymptotic distribution of the test statistics for cointegration depend upon the specification of the deterministic components, as demonstrated in Johansen (1994).

The estimate of β, $\widehat{\beta}$, is given by the r-largest eigenvectors associated with the eigenvalues $\widehat{\lambda}$. Hypothesis tests on $\widehat{\beta}$ can be conducted using likelihood ratio (LR) tests with standard χ^2 inference. Let the form of the linear restrictions on β be given by $\beta = H\varphi$ where H is a $p \times s$ matrix of restrictions and φ is a $s \times r$ matrix of unknown parameters. The LR test statistic is given by,

$$T \sum_{i=1}^{r} \ln \left[\frac{(1-\widetilde{\lambda}_i)}{(1-\widehat{\lambda}_i)} \right] \sim \chi^2_{r(p-s)} \tag{9}$$

where $\widetilde{\lambda}_i$ are the eigenvalues from the restricted MLE.

We also consider an additional test of the no cointegration null hypothesis that is systems based. Horvath and Watson (1995) develop a restricted version of the Johansen (1991) maximum likelihood systems based cointegration test. This test can be used to test the null hypothesis of no cointegration when the cointegrating vector is prespecified. This type of system based test may be more powerful than univariate tests, when there are no structural breaks.

Conventional tests for cointegration are usually conducted under the assumption that the long-run equilibrium between the variables in question is stable over the entire sample period. This may be a misleading assumption, especially if there were significant changes in the economic environment over the sample period. It is well known that structural breaks in time series can bias tests of unit roots and cointegration. As Canada has undergone changes in monetary policy as well as taxing policies during the sample period under study, it would seem prudent to investigate for any structural change. We conduct dynamic econometric specification analysis to investigate the existence of such regime changes and account for them in the estimation results. With regard to the stability of the cointegrating vector, Hansen and Johansen (1993) have developed a recursive test for the constancy of the cointegrating space within the FIML estimation procedure. The test is the likelihood ratio (LR) test of (9) calculated recursively over a sub-sample period. This procedure fixes the short-run dynamics at the full sample estimates and treats the full sample estimate of the cointegration vector as the null hypothesis in the recursive tests. This implies that rejections of stability are due to a change in the long-run relationship not due to shifting dynamics. Hansen and Johansen (1993) stability tests along with ocular inspection show that there were indeed structural breaks in the consumption/income ratio, the equity wealth/income ratio and in the linkage between the interest rate and inflation. We thus, augment our Johansen cointegration tests with dummy variables to take account regime shifts.

4 Data and Empirical Results

4.1 Description of Data

The data employed in this paper are obtained from the Bank of Canada and combined with wealth data developed by Macklem (1997). The data used in this paper are quarterly and measured in real per-capita terms. The GDP price deflator is used to covert nominal magnitudes into real variables. The variables include disposable income, total consumption expenditures, seasonally adjusted M1 monetary aggregate, Equity wealth, 3-month Treasury bill rate, and inflation computed as log differences of the GDP price deflator (annualized) over the 3-month maturity period of the Treasury bill. Nominal equity wealth is defined as the "current" value of shares held by persons and unincorporated businesses. Current value is measured as the sum of book value and cumulated retained earnings. We also investigate the effects of substituting non-durables and service consumption for total consumption and also investigate alternative measures of wealth.[3]

Figure 1 plots the six series. The upper left panel plots the natural (log) of real per-capita total consumption and (log) real per-capita disposable income. Total consumption and real disposable income for Canada move together upwards, suggesting a similar rate of growth. However, a careful examination reveals that the deviation between (log) income and (log) consumption increased during the 1980s, as consumption shifted downwards during the recession in the early 1980s and then resumed its long-run rate of growth. This is consistent with the findings of Lee and Siklos (1993), who document a sharp change in the consumption-income relationship occurring after the recession in Canada during the early 1980s. This result suggests a potential structural change in the consumption-disposable income relation. Because the variability of consumption spending differs between durables and non-durables and services, we test separately whether one of the major components of consumption is cointegrated with real disposable income. Since non-durables and services are the largest component of total consumption, results can be sensitive to how spending by consumers is aggregated. A graph of (log) real per-capita disposable income and (log) real per-capita non-durable and service consumption (not shown) reveals a similar relationship. We therefore focus on total consumption spending.

The upper right panel of Figure 1 plots the (log) of real per-capita equity wealth and (log) real per-capita consumption. Until the early 1980s equity wealth grows at a slower rate than consumption. The first time period in which the growth in equity wealth exceeds that of consumption growth is 1980Q1. During the post-1982 period the growth in equity wealth was persistently higher than the growth in consumption. The lower left panel of Figure 1 plots the M1 velocity against the 3-month Treasury bill interest rate. The lower right panel of Figure 1 plots the 3-month Treasury bill rate and the annualized inflation rate over the following three-months. We again see a change in the linkage of these variables developing during the post-1980 period. The inflation rate is persistently below the Treasury bill rate during the post-1980 period. In 1981Q3, the spread between the Treasury bill rate and the inflation rate reached a peak of 13.64%.

We can get a much sharper picture of these relations by looking at Figure 2. The data in Figure 2 are demeaned values. The upper left panel plots the (log) consumption to income ratio. The upper right panel plots the (log) consumption to equity wealth ratio. In 1980Q1, both the (log) consumption to income ratio and the (log) consumption to equity wealth ratio fall below their mean values. With some minor exceptions, during the post-1980 period, both the (log) consumption to income ratio and the (log) consumption to equity wealth ratio remain below their mean values. This reflects both a decrease in consumption and an increase in equity wealth. In the lower right panel of Figure 2 the de-meaned ex-post real interest rate (Treasury bill rate minus inflation) is plotted. The ex-post real interest rate remained, for the most part, below its mean (with some exceptions) during the pre-1980 period. In 1980Q4 the real interest rate rises above its mean, and remains above its mean thereafter, with some minor exceptions. In all cases it appears that there was a structural change that occurred in the consumption/income, consumption/wealth, and the real interest rate series somewhere between 1980 and 1982. The lower left panel of Figure 2 shows de-meaned deviations from a (log) money demand function with a unitary income elasticity and an interest elasticity of 0.12 in absolute value. (The interest rate is not logged in this relation.) This relation appears relatively stable with the exception of some large deviations during the late 1970s and early 1980s, corresponding to the contractionary monetary policy followed by the Central Bank of Canada during this time period.

Table 1 reports descriptive statistics on the levels (not in logs) of the consumption/income ratio, the consumption/equity wealth, the ex-post real interest rate, and deviations from (log) money demand with unitary income elasticity and interest elasticity of 0.12 in absolute value, for two time periods; 1964Q1-1981:Q4 and 1982:Q1-1994Q4. The consumption/income ratio fell from an average of 0.907 during the first sub-period to 0.866 during the second sub-period. These averages are statistically different from each other. The consumption/equity wealth ratio decreased substantially from an average of 1.248 during the first subperiod to 0.741 during the second sub-period. The ex-post real interest rate, which averaged only 0.334 percent during the first period increased to 6.278 percent during the second period. Deviations from the (log) money demand function appear to be similar during both time periods. Column 3 of Table 1 reports difference of means tests. These tests show that the mean values of the two sample periods are statistically significant from each other. These results suggest, with the exception of (log) money demand, dummy variables will have to be included in any cointegration analysis in order to account for the structural shifts in these variables. We turn to a multivariate analysis of these relations next.

4.2 Unit Root Tests

We begin our empirical analysis by examining the order of integration of our variables. Four are measured as the natural log of real per-capita values, i.e. total consumption spending (c), disposable income (y), equity wealth (w), and M1 money supply (m). The 3-month Treasury bill rate (i) and inflation rate(π) are measured at annual rates. Table 2 presents the results of the univariate unit root tests for the above six variables. Panel A displays the results for the variables that are demeaned by the GLS procedure. Panel B presents results for the data that are detrended by the GLS procedure. The first column of each panel lists the variable of interest. The second column of each panel displays the parameter indicating the degree of persistence ($\hat{\rho}$). All values of $\hat{\rho}$

are close or equal to one, with some slightly above one. Columns 3 and 4 present ADF and PP unit root test results. Column 5 displays the DF-GLS test proposed by Elliot et. al. (1996). Columns 6 and 7 report the results of the two M-tests.[4] Column 8 shows the lag truncation parameter chosen using the modified information criterion. As the ADF and PP tests yield results similar to the M-tests, there would appear to be little evidence of a large negative moving average component in our data. In no case can the null hypothesis of unit root be rejected.

4.3 Cointegration Tests Not Accounting For Regime Shifts

Table 3 reports multivariate cointegration tests. We begin by testing our four cointegrating relations wiyhout any dummy variables included to account for structural shifts in the relation. The Horvath-Watson test is used to test the null hypotheses of no cointegration against the alternative of a pre-specified cointegrating vector between c and y and c and w, where the cointegrating vector for both relations is specified to be [1, -1]. We also test the null hypothesis of no cointegration between i and π, against the alternative hypothesis of a specified cointegrating vector of [1, -1.6], as this is close to the estimated cointegrating vector. Finally we test the null hypothesis that there is no cointegration among m, y, and i, against the alternative that the specified cointegrating vector is [1,-1, 0.12]. In order to apply the Johansen tests, a lag length (k) in equation (8) must be chosen. The absence of serial correlation is a crucial condition, because if it is present, the trace tests can be oversized. We choose the smallest value of k for which the Ljung-Box Q-statistic indicates no serial correlation in the VECM of equation (8). We select the minimum lag length of which the residuals are white noise. Results for the Johansen test when no dummy variables are included (not reported to conserve space) and the Horvath and Watson (1995) statistic (reported in Column 5 of Table 3) do not find evidence of cointegration for the c-y or c-w relations, although cointegration is found for the money demand relation and Fisher relation. These findings are consistent with evidence in figure 2 and table 1.

The Horvath and Watson test results displayed in Column 5 of Table 3 are obtained under the assumption that the long-run equilibrium between c and y, and c and w, m, y, and i, and π are stable over the entire sample. This is a misleading assumption, especially if there were significant changes in the economic environment over the sample period, as is the case here. It is crucial to the subsequent dynamic analysis that the specification of the econometric model account for any such regime shifts.

4.4 Cointegration Tests Allowing For Regime Shifts

From Figure 2 we saw that there appeared to be at least two, and possibly three, structural changes that occurred in the long-run relationships considered. Figures 3 present the Hansen and Johansen (1993) recursive LR statistics for each cointegrating relation when regime shift dummy variables are included. These recursive LR statistics test for the stability of the cointegrating vector. These recursive statistics are obtained by treating the short-run dynamics as known and equal to the full sample estimates in the VECM. Hansen and Johansen (1993) have demonstrated that the LR tests are asymptotically distributed as $\chi^2(1)$ variates. The LR statistic tests the stability of the restricted cointegrating vector (given in Column 1 of Table 3) for the entire sample. The test statistics are normalized by the appropriate 5% critical values so that values greater than one imply significance (i.e. rejection of a stable cointegrating vector). If the inclusion of the dummy variables leads to a stable cointegrating relation, then the LR statistics should be below the normalized critical value of unity. We can see from Figures 3 that once we account for regime shifts, we cannot reject a stable cointegrating vector over the entire sample for all relations considered.[5]

Further evidence concerning the importance of accounting for the regime shifts in the four equilibrium relationships is provided in figures 4 and 5. Figure 4 displays each series against its estimated permanent component implied the common trends representation of the cointegrated VAR (see the discussion in section 4.6). These estimates are taken from a VAR that does not include the regime shift dummy variables. Figure 5 plots the transitory components of each variable from this "uncorrected" VAR. Note that each of the transitory components appears to be non-stationary (this is confirmed by formal unit root tests that are not presented for brevity) violating the concept of a transitory component.

Because we assume these break dates represent exogenous changes in the Canadian economy, dummy variables are used to capture their effects in the VECM. The first dummy variable (D73) takes on a value of zero through 1972Q4, after which it takes on a value of one. The second dummy variable (D80) takes on a value of zero through 1979Q4 and one thereafter. The third dummy variable (D82) takes on a value of zero through 1981Q4 and 1 thereafter.

Column 1 of Table 3 shows the hypothesized cointegrating vector and the specific dummy variables include in the VECM. Column 2 of Table 3 presents the Johansen-Jesulius (JJ) Trace statistic. Column 3 of Table 3 presents the estimated cointegrating vector (with dummy variables included). Column 4 of Table 3 presents a likelihood ratio test statistic, which tests whether one can reject the restricted cointegrating vector shown in Column 1 (with dummy variables included to allow for regime shifts). These tests show that when dummy variables are included in the cointegrating relations, the one-for-one linkage between (log) real per-capita consumption and (log) real disposable income as well as the one-for-one linkage between (log) real per-capita consumption and (log) real equity wealth cannot be rejected. When we substitute non-durable and service consumption expenditures in place of total consumption, we find similar results and are unable to reject the restriction.[6] However, when we include alternative measures of wealth, such as wealth excluding equity, we strongly reject this restriction and the coefficient on the wealth variables is of the wrong sign.

The above results imply that the (log) consumption/income ratio and the (log) consumption/wealth ratio are stable only after accounting for a shift in the mean of the process. Due to the decline in consumption and increase in wealth, these ratios have reached a new lower equilibrium level in the post-1980 period. The results also suggest that a stable money demand function exists over the entire period. A unitary income elasticity is found with an interest elasticity of 0.12 in absolute value. This result was also confirmed by a Fully-Modified OLS (see Phillips and Hansen, 1990) procedure in which the (log) velocity (y-m) is regressed on the 3-month nominal interest rate. The result of this estimation yielded an interest rate elasticity of 0.12 in absolute value. The specification of the deterministic components has important implications for the estimate of the Fisher effect, as well as any cointegrating relation. Crowder and Hoffman (1996), using U.S. data, found that when a superfluous non-zero mean is included in the specification, estimates of the Fisher effect will be biased. Because the Treasury bill rate and the inflation rate are non-trending variables, we constrain the constant term in the cointegrating vector to be zero. The estimated cointegrating vector between i and π is [1,-1.64]. We cannot reject the null hypothesis (at the 5% level) that the cointegrating vector between the 3-month Treasury bill rate and the inflation rate is [1,-1.6]. This estimate of the Canadian Fisher effect is within the range implied by the tax rates of Daly and Jung (1987). The LR test of the restriction that the cointegrating vector between the interest rate and inflation is [1,-1] is rejected and the recursive LR tests of this restriction show the vector to be unstable.

4.5 Multivariate Dynamic Analysis

It has been shown that when properly specified as a cointegrated system, accounting for regime shifts, the above four long-run relations in Canada are satisfied and are stable over the sample period. We can obtain insight into the short-run relationship between these variables from an examination of the estimates from the VECM of the full system, incorporating all four cointegrating vectors. Table 4 presents estimates of the full system VECM. Because the longest lag length observed in Table 3 was 5 (k =5 in equation 8), we employ this lag length in our system VECM. The residuals from each VECM were tested for serial correlation, normality and ARCH effects. An examination of the p-values on the Ljung-Box Q-statistics reveals no evidence of serial correlation in any equation. (see Column 6 of Table 4) The wealth equation residuals suffered from minor ARCH effects while the residuals of the money equation showed evidence of minor deviations from normality. Gonzalo (1994) has demonstrated, using Monte Carlo simulations, that the Johansen procedure performs well even when disturbances are non-spherical, that is, estimates and inferences on β are still valid. The inferences from the VECM, however, are affected. Therefore, the standard errors used to make inference in the full-system VECM have been adjusted using the Newey-West consistent covariance matrix with a Bartlett correction that includes three autocovariances in the calculation.

The error correction coefficients, α_j, in each equation measure the one-period response of each of the left-hand-side variables changes in the current period in response to a disequilibrium last period. For the system in Table 4, j=1 for the money demand CIV (y-m-0.12), j=2 for the consumption-income CIV (c-y), j=3 for the Fisher equation (i-1.6π) and j=4 for the consumption-wealth CIV (c-w).

The results of the full-system VECM (including dummy variables), reveals some interesting findings. First we observe that all the error-correction coefficients in both the income and inflation rate equation are insignificantly different from zero. This implies that disposable income and inflation are weakly exogenous. (See Engle, Hendry and Richard (1983)). In other words, when deviations from long-run equilibrium occur, it is real per-capita, equity wealth, consumption, money, and interest rate that adjust to restore long-run equilibrium. With respect to the interest rate equation, the positive coefficient on the second error-correction term implies that an increase in the consumption-income ratio causes interest rates to rise to restore equilibrium. The negative coefficient on the third error-correction coefficient implies that an increase in inflation causes nominal interest rates to rise to restore equilibrium. The negative coefficient on the fourth error-correction coefficient implies that an increase in the consumption-wealth ratio causes the nominal interest rate to fall to restore equilibrium. With respect to the consumption equation, the negative coefficient on the third error correction term implies that an increase in the nominal interest rate, or a fall in inflation (i.e. a rise in real interest rates) will cause consumption to fall. With respect to the wealth equation, the positive coefficient on the first error correction term implies that an increase in income, a fall in money demand, or a fall in the nominal interest rate will cause wealth to increase. The positive coefficient on the fourth error correction coefficient has the theoretically incorrect sign, as it implies that a decrease in consumption will cause wealth to fall. With respect to the money demand equation, the positive coefficient on the first error correction term implies that a rise in income will cause money demand to increase. The positive coefficient on the third error correction term implies that an increase in nominal interest rates or a decrease in inflation will cause money demand to increase.

Table 4 also reports F-tests for the joint significance of the lagged differenced terms in the VECM. With the exception of the F-test on the lagged changes of inflation in the inflation equation, the other F-tests in the income and inflation equations indicate that one cannot reject the null that the coefficients on the lagged changes are jointly zero. These results imply that income is strongly exogenous. If all of the F-tests indicate that none of the lagged changes of the independent variables are significant, then this implies that there is no short-run Granger causality running in the direction of the dependent variable. The significance of the error-correction term indicates long-run Granger causality.

4.6 Permanent and Transitory Decompositions

The Granger Representation Theorem implies that a p dimensional cointegrated vector I(1) process X_t may be expressed in terms of its permanent and transitory components as

$$X_t = X_0 + C(1)\left(\sum_{s=1}^{t} \varepsilon_s + \mu t\right) + C^*(L)\varepsilon_t \tag{10}$$

where $C(1)$ is the total impact of the accumulated errors, ε_s, on each variable in the system such that the trend is a pure random walk (with possible drift). The $C^*(L)$ is a matrix polynomial in the lag operator that captures the cyclical behavior of the variables in the system. Thus, equation (10) is simply a trend/cycle decomposition.

Johansen (1991) demonstrates that when the cointegration rank of r has been established, the impact matrix C(l) may be represented as

$$C(1) = \beta^*_\perp \alpha^{'}_\perp \tag{11}$$

where $\beta_\perp$ and $\alpha_\perp$, are p x $(p{-}r)$ matrices that are orthogonal complements to the p x r cointegration, β, and error correction, α, matrices, respectively. Johansen (1991) and King, et al. (1991) give $\alpha^{'}_\perp$, the interpretation as the common trends.

Each of the six variables in our system is decomposed into a permanent and transitory component. With six variables and four cointegrating vectors, there are two common trends among the six variables. As mentioned above, income and inflation were found to be weakly exogenous. This result implies that these variables represent the sources of the two common trends in the system. The accumulations of innovations to income form the first common trend while innovations to inflation form the second common trend. Note that the permanent components of each series are a weighted average of the two common trends except for the permanent component of real per-capita income and inflation because of the weak exogeneity restrictions that cannot be rejected by the data. The six series along with their associated permanent components are plotted in Figure 4 (for the case with no regime shifts) and Figure 6 (where regime shifts are accounted for by the inclusion of dummy variables). The transitory components (actual minus permanent component) of each series are plotted in Figure 5 (for no regime shifts) and Figure 7 (where regime shifts are accounted for by the inclusion of dummy variables).

We can see that there is a close linkage between the actual series and its corresponding permanent component once the regime shifts are included. There is a striking difference between Figure 4 and Figure 6. From Figure 6, one can see that the inclusion of the regime shift dummy variables results in a close relation between the actual series and its permanent component. From Figure 7 we can see that the transitory components are stationary, as they should be, once we account for the structural breaks in the processes.

4.7 Impulse Response Analysis

We have found evidence of cointegration consistent with the theoretical restrictions for four different relations. But evidence of cointegration in isolation does not tell the entire story. To be more specific, having knowledge of cointegration can help to explain the movements of these variables through time in response to discernible shocks. This provides insight on the relationship between the variables in the cointegrated system, not only over the long run, but over the short and intermediate terms as well.

Innovation analysis is conducted from the MA representation. In order to give the innovations or shocks a structural interpretation identifying restrictions must be imposed as in standard VAR analysis. Crowder (1995) demonstrates, using a bivariate cointegrated model, that identification of the structural permanent and transitory shocks is achieved by the assumption of independence between the innovations or shocks, which is a standard assumption in this literature. This section analyzes the dynamics of the system of four cointegrating vectors. The dynamic behavior of the statistical model is compared with the implications of the theoretical model using Impulse Response Functions (IRFs), which are calculated from the structural common trends representation.

Recall that income and inflation were found to be weakly exogenous in our VECM system of Table 4. This result implies that innovations to income and inflation accumulate to form the two common trends of the six variable system with four cointegrating vectors. In other words, when deviations from long-run equilibrium occur, it is real per-capita, equity wealth, consumption, money, and interest rate that adjust to restore long-run equilibrium. Given these results, we can interpret the first permanent shock as a real (productivity) shock and the second as a nominal (money growth) shock.

Figures 8 and 9 presents the cumulative impulse response of each series to a shock to the two permanent components while Figures 10 through 13 present the cumulative impulse response of each series to the four transitory components. From Figure 8 we can see that a one standard deviation shock to real income leads to an increase in real per-capita income, consistent with a real productivity shock. Contrary to theoretical expectations, a productivity shock leads to a fall in real interest rates. This results because the positive short-run response of inflation is larger than the negative short-run response of the short-term nominal interest rate. The response of real per-capita M1 is positive. Such dynamic behavior suggests that the central bank accommodates the productivity shock. We also estimate a smaller version of our model where we have a four variable VAR with y, π, i, and m. Here we have two long-run equilibrium relations; a money demand equation and a Fisher equation. Such a system was estimated by Crowder, Hoffman and Rasche (1999). We obtain the same results using the smaller system.

Figure 9 shows the response of our variables to a nominal money growth shock. These results are more in line with theory. A negative money growth shock causes real interest rates to rise in the short-run. This is the result of inflation falling in the short-run by a larger amount than short-term nominal interest rates fall. In addition, per-capita consumption and wealth also decline in the short-run, while money demand increases.

5 Summary and Conclusion

Three long-standing long-run equilibrium relationships examined in macroeconomics are the consumption-income relation, the money demand relation and the Fisher equation. A fourth linkage that also has some importance is the long-run linkage between wealth and income. Theory suggests that the above relations should have a long-run equilibrium stable relation over time. The Canadian economy suffered a severe recession in 1980 and 1982. In addition, the Canadian economy pursued tight monetary policy and expansionary fiscal policy during the post-1980 period. In the post-1980 period Canada also increased incentives to save through changes in the tax laws. This paper applies modern time series methods to Canadian quarterly data from 1964Q1 to 1994Q4 to investigate the effect of these events on the four long-run relations mentioned above. The results of this paper show that, while the long-run linkage between these relationships was maintained after 1982, permanent downward level shifts occurred in the consumption/income ratio, and the wealth/income ratio while there was an increase in ex-post real interest rates. The demand for M1 function was found to be stable over the entire sample period examined, with no level shifts. This paper finds considerable support that four traditional long-run economic relations in Canada are stable over the period 1964Q1-1994Q2, when regime shifts dummy variables are included to account for structural breaks.

The above results are also consistent with the literature suggesting that the Central Bank of Canada maintained a high level of interest rates to keep inflation low, which they were successful in doing.

Permanent and transitory decomposition and impulse response analysis is also conducted.

Notes

[1]The results of the cited studies are dependent on the form of the restrictions used to identify the structural relationships. Sims (1980) has argued that it is difficult to justify short-run restrictions. And while long-run restrictions have more theoretical appeal, they may not adequately identify the model as demonstrated by Faust and Leeper (1994).

[2]The restriction that rules out processes integrated of orders higher than one is given in Johansen (1991). Specifically the matrix $\alpha'_{\perp}(I_p - \sum_{j=1}^{k-1} \Gamma_j)\beta_{\perp}$ must be non-singular.

[3]We would like to thank Tiff Macklem for providing data on various measures of wealth. For a more detailed discussion of data sources see Macklem (1997).

[4]Serena Ng kindly provided the GAUSS code used to implement these tests.

[5]An alternative test for parameter non-constancy for I(1) variables is developed in Hansen (1992). Hansen proposes three tests (SupF, MeanF, and Lc). All of these have the null hypothesis of parameter constancy but differ in their alternatives. The SupF test tests for a structural break of unknown timing and is appropriate for determining if there has been an abrupt shift in regime. The MeanF and Lc model the parameters as a martingale under the alternative. Hence, any change is viewed as a gradual process.

[6]Cointegration tests (including D80, and D82) between consumption of non-durables and services and disposable income yields trace statistics of 20.14 and 6.50, with a cointegrating vector of [1,-0.95]. This provides evidence of one cointegrating vector. Cointegration tests (including D73, D80, and D82) between consumption on non-durables and services and equity wealth yield trace

statistics of 11.01 and 3.18, with a cointegrating vector of [1, -0.48]. These trace statistics indicate that the null of no cointegration between consumption on non-durables and services and equity wealth cannot be rejected.

References

[1] Albert Ando and Franco Modigliani. The Life-Cycle Hypothesis of Saving: Aggregate Implications and Tests. *American Economic Review*, 53:55–84, March 1963.

[2] Olivier J. Blanchard. A Traditional Interpretation of Macroeconomic Fluctuations. *American Economic Review*, 79:655–673, 1989.

[3] Olivier J. Blanchard and Danny Quah. The Dynamic Effects of Aggregate Demand and Supply Disturbances. *American Economic Review*, 79:655–673, 1989.

[4] Paul M. Boothe and Stephen S. Poloz. Unstable Money Demand and the Monetary Model of the Exchange Rate. *Canadian Journal of Economics*, 21(4):785–798, November 1988.

[5] Carolyn Clark. The Demand for Money and the Choice of a Permanent Income Estimate: Some Canadian Evidence, 1926-65. *Journal of Money, Credit and Banking*, 5(3):773–793, August 1973.

[6] Thomas F. Cooley and Stephen F. LeRoy. Atheoretical Macroeconometrics: A Critique. *Journal of Monetary Economics*, 16:283–308, 1985.

[7] William J. Crowder. The Dynamic Effects of Aggregate Demand and Supply Disturbances: Another Look. *Economics Letters*, 49(3):231–237, September 1995.

[8] William J. Crowder. The Long-Run Fisher Relation in Canada. *Canadian Journal of Economics*, 30(4):1124–1142, November 1997.

[9] William J. Crowder and Dennis L. Hoffman. The Long-Run Relation Between Nominal Interest Rates and Inflation: The Fisher Equation Revisited. *Journal of Money, Credit and Banking*, 28(1):102–118, February 1996.

[10] William J. Crowder, Dennis L. Hoffman, and Robert H. Rasche. Identification, Long-run Relations, and Fundamental Innovations in a Simple Cointegrated System. *Review of Economics and Statistics*, 81(1):109–121, February 1999.

[11] M.J. Daly and J. Jung. The Taxation of Corporate Investment Income in Canada: An Analysis of Marginal Effective Tax Rates. *Canadian Journal of Economics*, 20:555–588, 1987.

[12] S. D. Dutt and D. Ghosh. The Fisher Hypothesis: Examining the Canadian Experience. *Applied Economics*, 27:1025–1030, 1995.

[13] Graham Elliot, Thomas J. Rothenberg, and James H. Stock. Efficient Tests for an Autoregressive Unit Root. *Econometrica*, 64(4):813–836, July 1996.

[14] Robert F. Engle, David F. Hendry, and Jean-Francois Richard. Exogeneity. *Econometrica*, 51(2):277–304, March 1983.

[15] Jon Faust and Eric Leeper. When do long-run identifying restrictions give reliable results? *Journal of Business and Economic Statistics*, 15:345–53, July 1997.

[16] David K. Foot. The Demand for Money in Canada: Some Additional Evidence. *Canadian Journal of Economics*, 10(3):475–485, August 1977.

[17] Milton Friedman. *A Theory of the Consumption Function*. Princeton University Press, Princeton, New Jersey, 1957.

[18] Jordi Gali. How Well Does the IS-LM Model Fit Postwar U.S. Data? *Quarterly Journal of Economics*, 107, 1992.

[19] Jesus Gonzalo. Five Alternative Methods of Estimating Long-Run Equilibrium Relationships. *Journal of Econometrics*, 60(1-2):203–233, Jan.-Feb. 1994.

[20] Bruce H. Hansen. Tests for Parameter Instability in Regressions with I(1) Processes. *Journal of Business and Economic Statistics*, 10:321–335, 1992.

[21] Henrik Hansen and Soren Johansen. Recursive Estimation in Cointegrated VAR-Models. *University of Copenhagen*, Working Paper, January 1993.

[22] Alfred A. Haug and Robert F. Lucas. Long-Run Money Demand in Canada: In Search of Stability. *Review of Economics and Statistics*, 78:345–348, May 1996.

[23] Dennis L. Hoffman, Robert H. Rasche, and Margie A. Tieslau. The Stability of Long-run Money Demand in Five Industrialized Countries. *Journal of Monetary Economics*, 35, 1995.

[24] Michael T. K. Horvath and Mark W. Watson. Testing for Cointegration When Some of the Cointegrating Vectors are Prespecified. *Econometric Theory*, 11(5):984–1014, December 1995. Working Paper No. 15, Research Department, Federal Reserve Bank of Chicago.

[25] Søren Johansen. Estimation and Hypothesis Testing of Cointegration Vectors in Gaussian Vector Autoregressive Models. *Econometrica*, 59:1551–1580, 1991.

[26] Søren Johansen. The Role of the Constant and Linear Terms in Cointegration Analysis of Non-Stationary Variables. *Econometric Reviews*, 13(2):205–230, July 1994.

[27] Robert G. King, Charles I. Plosser, James H. Stock, and Mark W. Watson. Stochastic Trends and Economic Fluctuations. *American Economic Review*, 81:819–840, 1991.

[28] Edward E. Leamer. Vector Autoregressions for Causal Inference? *Carnegie-Rochester Conference Series on Public Policy*, 22(0):255–303, Spring 1985.

[29] Hahn S. Lee and Pierre L. Siklos. The influence of seasonal adjustment on the canadian consumption function, 1947-1991. *Canadian Journal of Economics*, 26(3):575–89, August 1993.

[30] Sydney Ludvigson and Charles Steindel. How Important Is the Stock Market Effect on Consumption? *Federal Reserve Bank of New York Economic Policy Review*, 5(2):29–51, July 1999.

[31] James G. MacKinnon, Alfred A. Haug, and Leo Michelis. Numerical Distribution Functions of Likelihood Ratio Tests for Cointegration. *Queen's University*, Working Paper, September 1996.

[32] R. Tiff MacKlem. Aggregate Wealth in Canada. *Canadian Journal of Economics*, 30(1):152–168, February 1997.

[33] Ronald McDonald and P. D. Murphy. Testing for the Long-Run Relationship Between Nominal Interest Rates and Inflation using Cointegration Techniques. *Applied Economics*, 21:439–447, 1989.

[34] Frederic S. Mishkin. The Real Interest Rate: A Multi-Country Empirical Study. *Canadian Journal of Economics*, 17(2):283–311, May 1984.

[35] Serena Ng and Pierre Perron. Unit Root Tests in ARMA Models with Data-Dependent Methods for the Selection of the Truncation Lag. *Journal of the American Statistical Association*, 90(429):268–281, March 1995.

[36] Serena Ng and Pierre Perron. Lag Length Selection and the Construction of Unit Root Tests with Good Size and Power. Boston College, Department of Economics Working Paper 369, September 2000.

[37] Michael Osterwald-Lenum. A Note With Quantiles of the Asymptotic Distribution of the Maximum Likelihood Cointegration Rank Test Statistics. *Oxford Bulletin of Economics and Statistics*, 54:461–471, 1992.

[38] Sastry G. Pantula. Asymptotic Distributions of Unit-Root Tests When the Process Is Nearly Stationary. *Journal of Business and Economic Statistics*, 9(1):63–71, January 1991.

[39] Pierre Perron and Serena Ng. Useful Modifications to Some Unit Root Tests with Dependent Errors and Their Local Asymptotic Properties. *Review of Economic Studies*, 63(3):435–63, July 1996.

[40] Peter C.B. Phillips and Bruce H. Hansen. Statistical Inference in Instrumental Variables Regression with I(1) Processes. *Review of Economic Studies*, 57:99–125, 1990.

[41] Peter C.B. Phillips and Pierre Perron. Testing for a Unit Root in Time Series Regression. *Biometrika*, 75:335–346, 1988.

[42] Said E. Said and David A. Dickey. Testing for Unit Roots in Autoregressive–Moving Average Models of Unknown Order. *Biometrika*, 71:599–607, 1984.

[43] G. William Schwert. Effects of Model Specification on Tests for Unit Roots in Macroeconomic Data. *Journal of Monetary Economics*, 20:73–103, 1987.

[44] G. William Schwert. Tests for Unit Roots: A Monte Carlo Investigation. *Journal of Business and Economic Statistics*, 7:147–159, 1989.

[45] Christopher A. Sims. Macroeconomics and Reality. *Econometrica*, 48:1–48, 1980.

[46] Tony S. Wirjanto. Aggregate Consumption Behaviour and Liquidity Constraints: The Canadian Evidence. *Canadian Journal of Economics*, 28(4b):1135–52, November 1995.

Table 1
Descriptive Statistics of Equilibrium Ratios

Equilibrium Relation	Mean: 1964:1-1981:4	Mean: 1982:1-1994:2	Difference in Means Test
Consumption/Income Ratio	0.907 (0.0019)	0.866 (0.0027)	z=12.42
Consumption/Wealth Ratio	1.248 (0.0339)	0.741 (0.0208)	z=12.74
Ex-post Real Interest Rate	0.334 (0.4676)	6.278 (0.3513)	z=10.16
Money Demand	−0.984 (0.0370)	−1.179 (0.0425)	z=3.46

NOTES: Numbers in brackets represent standard error. The z-statistic is distributed N(0,1). The z-statistic is calculated as the absolute value of the difference in the two means divided by the square root of the sum of the squared standard errors.

Table 2
Unit Root Test Statistics

Panel A							
Variable	$\hat{\rho}$	$ADF{-}t_{\hat{\beta}}$	$PP{-}Z_{\rho}$	DF^{GLS}	MZ_{ρ}	MZ_t	k
Consumption	1.002	-1.013	-3.630	0.563	0.561	0.491	12
Income	1.000	-1.065	-2.814	0.176	0.289	0.225	11
Equity Wealth	1.001	-1.013	-3.630	0.563	0.561	0.491	12
Money	0.971	-1.181	-3.537	-1.169	-3.343	-1.503	0
Interest rate	0.970	-2.144	-8.424	-1.421	-4.113	-1.429	2
Inflation	0.899	-1.607	-9.264	-1.616	-5.901	-1.688	4
5% Critical Value		-2.86	-14.1	-1.98	-8.10	-1.98	
Panel B							
Consumption	0.972	-1.646	-6.911	-1.482	-12.754	-2.395	12
Income	0.987	-0.438	-0.998	-0.491	-1.166	-0.416	1
Equity Wealth	0.921	-2.570	-12.932	-2.582	-12.842	-2.528	1
Money	0.971	-0.987	-3.248	-1.175	-3.487	-1.153	0
Interest rate	0.936	-2.041	-9.785	-1.996	-9.074	-2.028	2
Inflation	0.885	-2.000	-11.262	-1.794	-6.467	-1.790	4
5% Critical Value		-3.41	-21.00	-2.91	-17.30	-2.91	

Note:
Panel A: The *ADF* and *PP* test results are from a model that was *OLS* demeaned The other test results are from a model that was demeaned by the *GLS* procedure.
Panel B: The *ADF* and *PP* test results are from a model that was *OLS* detrended. The other test results are for the data that was detrended by the *GLS* procedure.
The parameter $\hat{\rho}$ is the autoregressive coefficient. The parameter, k, is the lag length.

Table 3
Multivariate Unit Root Test Statistics

Variables [*Hypothesized CIVs*] *Significant Dummy Variables*	*Johansen Trace Statistics* VAR Lag Length	*Estimated CIVs*	*Tests of Restricted CIVs*	*Horvath-Watson Tests*
c_t, y_t $[1, -1]$ $D82$	r = 0 15.46** r ≤ 1 6.63 k = 2	[1, -1.05]	0.23 [0.64]	10.61 $L = 1$
c_t, w_t $[1, -1]$ $D73, D80, D82$	r = 0 13.44* r ≤ 1 3.55 k = 5	[1, -0.73]	0.22 [0.64]	9.81 $L = 1$
$m_t - p_t, y_t, i_t$ $[1, -1, 0.12]$ *None*	r = 0 38.73** r ≤ 1 15.11 k = 4	[1, -1, 0.13]	0.01 [0.99]	30.20** $L = 3$
i_t, π_{t+1} $[1, -1.6]$ *None*	r = 0 7.75* r ≤ 1 1.52 k = 5	[1, -1.64]	0.01 [0.99]	16.96** $L = 4$

Notes: ** and * designate significance at the 5% and 10% level, respectively. Numbers in brackets are p-values.
a. Column 1 displays the variables in the cointegrating relation, the hypothesized cointegrating vector, and the statistically significant dummy variables included in the relevant VECM equation. D73, D80, and D82 are dummy variables that take on the following values: D73=1 for the period 1973:Q1-1994:Q2, zero elsewhere; D80=1 for the period 1980:Q1-1994:Q2, zero elsewhere; D82=1 for the period 1982:Q1-1994:Q2, zero elsewhere.
b. Column 2 reports the Johansen (1991) trace statistics of the null of no cointegration. Critical values are from Osterwald-Lenum (1992).
The value of k is the number of lags in the levels VAR and k-1 in the differenced VECM and was chosen as the minimum lag necessary to yield statistically uncorrelated residuals.
c. Column 3 reports the estimated coefficients of the cointegrating vector with dummy variables included where appropriate.
d. Column 4 reports the Johansen -Jesulius χ^2 likelihood ratio test of the restricted cointegrating vector hypothesized in column 1. Marginal significance levels are given in brackets.
e. Column 5 presents the Horvath and Watson (1995) one-side (upper-tail) test of the null of no nointegration vs. the alternative of cointegration using the pre-specified cointegrating vector given in column one (no dummy variables included).
L is the lag length in the H-W model that is chosen as the lag where the BIC is minimized.

Table 4

Vector Error Correction Model Results

Equation	α_1	α_2	α_3	α_4	Q(30)	F-Δy_{t-i}	F-$\Delta \pi_{t-i}$	F-Δi_{t-i}	F-Δc_{t-i}	F-Δw_{t-i}	F-Δm_{t-i}
Δy	−0.007	0.181	0.000	0.017	27.87	1.09	1.34	0.45	0.83	0.94	2.08
	(0.009)	(0.102)	(0.000)	(0.014)	[0.58]	[0.37]	[0.26]	[0.77]	[0.50]	[0.44]	[0.09]
$\Delta\pi$	1.142	−20.336	0.131	−1.700	19.31	0.46	4.80	1.07	1.51	0.62	1.63
	(1.092)	(20.469)	(0.070)	(2.890)	[0.94]	[0.76]	[0.02]	[0.38]	[0.21]	[0.65]	[0.17]
Δi	−0.941	22.001**	−0.068*	−2.524**	26.99	3.00	2.68	3.93	0.53	1.48	0.17
	(0.559)	(9.233)	(0.034)	(1.022)	[0.62]	[0.02]	[0.03]	[0.01]	[0.71]	[0.22]	[0.95]
Δc	0.020**	−0.074	−0.0006**	0.003	31.87	2.45	2.20	0.51	3.29	1.83	0.44
	(0.007)	(0.076)	(0.0003)	(0.007)	[0.37]	[0.05]	[0.08]	[0.73]	[0.02]	[0.13]	[0.78]
Δw	0.130**	−0.960	0.004	0.317**	25.99	2.11	1.96	2.27	0.32	0.99	1.04
	(0.055)	(0.739)	(0.003)	(0.067)	[0.68]	[0.08]	[0.11]	[0.07]	[0.87]	[0.42]	[0.39]
Δm	0.07**	−0.109	0.0014*	0.035	30.07	0.758	1.37	0.78	0.35	1.26	3.95
	(0.013)	(0.168)	(0.0007)	(0.016)	[0.46]	[0.56]	[0.35]	[0.54]	[0.84]	[0.29]	[0.01]

NOTES:
The columns α_j correspond to the estimate of the error correction coefficient associated with each equation in the VECM.
α_j for j= 1, 2, 3, 4 correspond to cointegrating vector (CIV) j= 1,2,3, and 4
CIV_1 = y-m-0.12i; CIV_2 = c-y; CIV_3 = i-1.6π; CIV_4 = c-w
Numbers in **parentheses** are Newey-West corrected standard errors. Numbers in **brackets** are marginal significance levels. The lag length, k, used in the levels VAR is equal to 5 (4 difference terms in the VECM).
The variables, y, π, i, c, w, and m, are natural log of real per-capita disposable income, inflation over the 3-month maturity of the T-bill, 3-month Treasury bill, real per-capita total consumption expenditures, real per-capita equity wealth, and real per-capita M1 money supply.
** and * designate significance at 5% and 10% level respectively.
D73, D80, and D82 are dummy variables which take on the following values.
D73=1 for 1973Q1-1994Q2, 0 elsewhere;
D80-1 for 1980Q1-1994Q4, 0 elsewhere;
D82=1 for 1982Q1-1994Q2, 0 elsewhere.