

Domestic-Foreign Interest Rate Differentials:
Near Unit Roots and Symmetric Threshold Models

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Abstract

This paper investigates the near unit root behavior of interest rate differentials across countries using a symmetric-TAR process. We find that the time series properties of short-term interest differentials between the France, Germany, UK and US can be characterized by a symmetric-TAR process, which can explain its (near) unit root behavior. Results significantly reject a linear model in favor of the alternative hypothesis of a two-regime symmetric threshold model that exhibits significantly greater persistence within the threshold bands than when outside the threshold bands. Changes in the level of the US interest rate act as the trigger to switch from one regime to the other. We also show that a TAR modeling framework for interest rate differentials is able to yield the same results for the drift and volatility that other papers have reported using regime switching models for the level of the short-term interest rate.

1. Introduction

Many economic time series are strongly autocorrelated and can be modeled as linear (near) unit root or $I(1)$ processes. One such series is the interest rate differential. Crowder (1995), for example, shows that interest rate differentials can be characterized by a high degree of persistence and conditional heteroscedasticity. He finds that in most cases the null of unit root for interest rate differentials across countries cannot be rejected. This creates an inconsistency as uncovered interest parity (UIP) states that the interest differential (which is a very persistent series) is equal to the expected change in spot exchange rate (a series that exhibits virtually no serial correlation and is quite volatile). The mismatch between the time series properties of the change in the exchange rate (a stationary process) and the interest rate differential (a very persistent near unit root process) should create investment opportunities, and a substantial literature has emerged offering various explanations for this anomaly (see Engel, 1996, for a survey).

Our contribution is to apply a heteroscedastic band Threshold Autoregression (TAR) framework to model the interest rate differential between the US and Germany, the US and France, and the US and the UK. The interest rate differential between two countries is an important variable in international finance and a critical component of uncovered interest parity (UIP). The correct specification of the conditional distribution of interest rate differentials is one important factor towards understanding the failure to find support for UIP reported in most of the extant literature. Previous research explained this failure due to risk premia. Our paper shows that the time series properties of interest differentials exhibit significant TAR nonlinearities that can characterize the (near) unit root behavior of the interest rate differential reported in the extant literature. Further, we show that these nonlinearities are occurring from the innovations

in changes in the levels of domestic interest rates (e.g., US rate) that do not simultaneously match the time series properties of the foreign interest rate (e.g., German rate).

Studies by Caner and Hansen (2001), Enders and Granger (1998), Gospodinov (2005), Gonzalo and Gonzales (1999), and Lanne and Saikkonen (2002), argue that the apparent (near) unit root behavior of many financial and economic time series may be the result of omitted nonlinearity. For example, small deviations from a long-run equilibrium relationship may not be mean reverting due to the existence of transaction costs. However, once these deviations become large enough, arbitrage profits will exceed transaction costs and these deviations will revert to the long-run equilibrium. Taylor (2001) demonstrates if the true process is of a nonlinear type similar to that described above, then linear tests will fail to support mean reversion. For example, if the autoregressive representation of the interest rate differential switches between stationary and nonstationary regimes, then unit root testing procedures will have difficulty in detecting mean reversion, and false inference will occur due to size distortions induced by the misspecification.

The estimation of regime switching (RS) models to characterize the movements of economic variables that exhibit near unit behavior has become increasingly popular in recent years. One type of regime switching model that has become popular is the Threshold Autoregressive (TAR) Model. In the international finance arena, research has employed the TAR family of models in an attempt to model the so-called purchasing power parity (PPP) puzzle in which real exchange rates exhibit short-run fluctuations and, in addition, deviations from PPP have very long half lives.¹ The extant literature on the term-structure of interest rates frequently employs regime switching (RS) models as well (see Hamilton 1988; Sola and Driffill 1994; Bekaert, Hodrick and Marshall 2001). Regime switching models have also been applied

to the level of interest rates (Gray 1996; Ang and Bekaert 2002a). One attraction of RS models is that they can accommodate some of the nonlinearities in interest rates that appear in higher order unconditional moments. For example, Gray 1996 and Bekaert, Hodrick, Marshall, (2001) show that it is only at low levels of interest rates that interest rates behave as a random walk, whereas high levels of interest rates exhibit considerable mean reversion.²

In the above mentioned literature, there is no way to distinguish between nonlinearity and/or nonstationarity. In much of this literature the econometric techniques employed assume stationarity prior to fitting a nonlinear model. Thus, these studies are not able to reconcile the nonlinear adjustment that they find with the evidence of unit root behavior reported in empirical studies employing linear models. In this paper we circumvent these problems by employing tests and distribution theory developed by Caner and Hansen (2001) and Gospodinov (2005) that allow for the joint testing of nonlinearity and nonstationarity. This approach employs a symmetric TAR model with an autoregressive (AR) unit root that allows for an inner band of inaction where small deviations are not reversed and also captures large deviations outside the band of inaction which do exhibit mean reverting behavior.

Gonzalez and Gonzalo (1999) and Caner and Hansen (2001) have considered TAR models as alternatives to linear (near) unit root models. Their model allows for a series to have a (near) unit root in one regime while being stationary in the other. If the first order autoregressive coefficient is switching between two regimes, one that is stationary and the other non-stationary (or near unit root), then linear testing procedures will have difficulties detecting the mean reversion of the process. Caner and Hansen (2001) also derive statistical tests for testing their TAR models against linear (near) unit root models. These linear models assume a stationary threshold variable, which in practice is typically the lagged difference of the series. Lanne and

Saikkonen (2002) consider TAR models that have constant stable roots smaller than unity in all regimes and in which only the intercept term switches between different regimes.

A central difficulty in modeling interest rates and interest differentials (e.g. at weekly or monthly frequencies) is that innovations are highly persistent and exhibit strong conditional heteroskedasticity. One approach is to employ nonparametric methods to estimate the drift and diffusion functions and construct tests for nonlinearity. The nonparametric procedures suggested in the recent literature (Alt-Sahalia 1996; Stanton 1997), however, are not appropriate for highly persistent series and can lead to severe size distortions (Pritsker 1998 and Conley et. al. 1997) and spurious results (Chapman and Pearson 2000). These estimators are biased in the extremes of the estimated function where there are only a few observations available. In addition, they depend crucially on the choice of the bandwidth (smoothing) parameter. For highly persistent data, the normal recommendations for an optimal bandwidth are not appropriate. Furthermore, Mark and Moh (2002), examine UIP and the forward premium anomaly. Their results find ARCH effects and attribute them to central bank intervention. They also provide evidence for heavy tailed distributions in interest rate differentials and attribute this to “Big News.”

The methodology used in the current paper applies and extends the framework of Gospodinov (2005), who models the level of short-term interest rates as a threshold process and accommodates for a heteroscedastic error process. In this paper we modify this approach to allow for a symmetric heteroscedastic TAR process and investigate interest rate differentials between the US, UK, Germany, and France. Specifically, we allow for a central band within which the interest rate differential is (near) unit root and mean reverting outside the central band.³ The tests developed by Enders and Granger (1998) are unit root procedures that evaluate the null of no cointegration and requires that the cointegrating vector be (1,-1). In contrast, the

tests developed in Enders and Siklos (2001) are designed to be applied to the residuals from a linear cointegrating regression where the cointegrating coefficient is estimated. Neither of these tests allow for a central band of $I(1)$ behavior as is done in this paper. Additionally, given that Ang and Bekaert (2002b) show that regime switching models can model the non-parametric estimates of drift and volatility of short-term interest rates very well, a second objective of this paper is to examine whether TAR models can duplicate this feature (reported for the level of short rates) for the domestic-foreign interest rate differential.⁴

As a preview of our results, we find that TAR models can capture some of the important properties of movements in interest rate differentials over time. In particular, we find that interest rate differentials exhibit faster mean reversion outside of a symmetric band. Within this band, interest rate differentials are much more persistent and follow a near unit root process. In addition, we find that a TAR-GARCH(1,1) modeling framework can well approximate the non-parametric estimate of the drift and volatility of the interest rate differential. Results illustrate that a TAR model can replicate the drift and volatility functions that other papers have reported using Markov-Switching models for the level of the short-term interest rate.

The remainder of this paper is organized as follows: Section 2 presents the TAR modeling framework and some background literature. Section 3 presents the empirical results and section 4 concludes the paper.

2. Methodology and Background

2A. The TAR Model and Testing For Unit Roots

The threshold like behavior of the interest rate differential may arise from the presence of transaction costs that investors face (Anderson (1997)). Deviations from the arbitrage condition

will be temporary and the adjustment infrequent, triggered back to equilibrium when the differential increases above a certain threshold or is induced to move through actions taken by the central bank. In such models, arbitrage is not profitable if price discrepancies or interest rate differentials do not exceed transaction costs, giving rise to a “band of inactivity,” where deviations are not corrected. However, “large” price (interest rate) discrepancies that exceed transaction costs are quickly corrected, so that the differentials are driven back inside the band of inactivity where arbitrage is no longer profitable. The existence of transaction costs is not the only possible explanation for such behavior. For example, during periods of uncertainty investors may take little action and hence small interest rate differentials may persist for long periods of time. However, when deviations become large, investors become more confident about their forecasts of future events and take actions to correct large deviations in price or interest rate differentials. Balke and Wohar (1998), employing a stationary TAR model, find that the time series properties of covered interest parity are affected by thresholds, where arbitrage costs imply faster reversion the larger the innovation is outside the threshold bands. Furthermore, Saikkonen and Ripatti (1999) recently suggested that the Threshold Autoregressive (TAR) model is the appropriate framework for testing and modeling expectational economic relationships in the presence of “peso effects”. Obstfeld and Taylor (1997) also demonstrate that the TAR framework can model the PPP relationship.

As mentioned above, conventional unit root tests are unable to reject the unit root null for the interest rate differential across countries. These methods are not able to disentangle non-stationarity from non-linearity because of the joint modeling problem of unit roots and thresholds. If interest rates possess nonlinearities that differ across economies, and transactions costs imply a band of arbitrage inaction (allowing these nonlinearities to persist), the interest rate

differential may also possess a threshold. In addition, changes in business cycle conditions and monetary policy may cause real interest rates and expected inflation to behave differently during different time periods. If business cycles and monetary policy responses are nonsynchronous, the interest differential may also be effected. In this paper we employ a TAR framework to model the (near) unit root behavior, strong persistence, and a heteroscedastic error process characteristic of domestic-foreign interest rate differentials.

The original TAR models maintained the assumption that the data follow a stationary process. These modeling frameworks were not able to discriminate between non-stationarity and non-linearity. Caner and Hansen (2001) develop a framework that allows for both stationary and non-stationary processes. Their model permits all the slope coefficients to switch between the two regimes, and use both Wald and t-statistics to test differences in coefficients. To calculate the p-values, they use bootstrapping since their sampling distributions are non-standard due to the presence of possible unidentified parameters and non-stationarity. The basic TAR model allows the degree of autoregressive decay to depend on the state of the variable of interest. The so called M-TAR model, used by Enders and Granger (1998) and Caner and Hansen (2001) allow a variable to display different amounts of autoregressive decay depending on whether previous change is positive or negative.

Consider the following representation of a TAR process for a series y_t .

$$\Delta y_t = I_t \rho_1 y_{t-1} + (1 - I_t) \rho_2 y_{t-1} + \varepsilon_t, \quad (1)$$

where I_t is the Heaviside indicator function such that

$$I_t = \begin{cases} 1 & \text{if } y_{t-1} < \lambda \\ 0 & \text{if } y_{t-1} \geq \lambda \end{cases}$$

where λ = the value of the threshold and $\{\varepsilon_t\}$ is a sequence of zero-mean, constant-variance

iid random variables, such that ε_t is independent of $y_j, j < t$. Petrucelli and Woolford (1984) showed that the necessary and sufficient conditions for the stationarity of $\{y_t\}$ are $\rho_1 < 0, \rho_2 < 0$ and $(1 + \rho_1)(1 + \rho_2) < 1$ for any value of λ . When $\rho_1 = \rho_2$, the above is referred to as a symmetric TAR model. Tong (1983, 1990) showed that the least squares estimates of ρ_1 and ρ_2 have asymptotic multivariate normal distribution.

In testing for unit root behavior, the null and alternative hypotheses associated with (1) are given by

$$H_0 : \rho_1 = \rho_2 = 0.$$

$$H_1 : \rho_1 < 0 \text{ and } \rho_2 < 0.$$

where, (ρ_1, ρ_2) are the coefficients associated with y_{t-1} , (where the regression in (1) is often augmented with lagged difference terms) in regime one and two, respectively. When the above null cannot be rejected, the series y_t is an I(1) process, and y_t can be rewritten as a stationary threshold autoregression using differenced data. (Chan and Tong, 1985). Caner and Hansen (2001) suggest a third intermediate case:

$$H_2 : \begin{cases} \rho_1 < 0 & \text{and} & \rho_2 = 0, \\ & \text{or} & \\ \rho_1 = 0 & \text{and} & \rho_2 < 0. \end{cases}$$

If H_2 holds, the variable follows a partial unit root process, where it is unit root process in one regime, and mean-reverting the second regime. The existence of heteroscedasticity significantly complicates the derivation of the limiting distribution of the test statistics.

2B. Heteroscedasticity.

Gospodinov (2005) extends Caner and Hansen (2001) by allowing for conditional

heteroscedasticity of unknown form and a GARCH(1,1) error process. He uses bootstrap methods to obtain the nonstandard distributions. The following discussion draws heavily from Gospodinov. Assume a data generating process (DGP) given by the TAR model

$$y_t = I_{\{z_{t-d} < \lambda\}}(a_1 + b_1 y_{t-1}) + I_{\{z_{t-d} \geq \lambda\}}(a_2 + b_2 y_{t-1}) + \varepsilon_t \quad (2)$$

where $I_{\{q\}}$ is the indicator (Heaviside) function, z_{t-d} is a threshold variable, d is the delay parameter and λ is the threshold. Gospodinov (2005) is interested in the case when the largest AR root is close to one so he rewrites the above model as

$$\Delta y_t = \mu + \rho y_{t-1} + I_{\{z_{t-d} \geq \lambda\}}(\gamma + \phi y_{t-1}) + \varepsilon_t, t = 1, \dots, T, \quad (3)$$

where $I_{\{q\}}$ is the indicator (Heaviside) function, z_{t-d} is a threshold variable, d is the delay parameter, λ is the threshold, $\rho = b_1 - 1$, $\phi = b_2 - b_1$, $\mu = a_1$ and $\gamma = a_2 - a_1$. Reparameterizing the coefficient on y_{t-1} as local-to-zero $\rho = c/T$, where $c \leq 0$ is a constant, is very helpful in analyzing the properties of the estimators and the corresponding test statistics. Let $\theta_1 = (\mu, \rho)'$ and $\theta_2 = (\gamma, \phi)'$ denote the parameter vectors in regime 1 and 2 respectively and $\theta = (\theta_1', \theta_2')$. The null hypothesis is $H_0 : \theta_2 = 0$.

Equation (3) uses a dummy variable approach to test differences in the two regimes. If the null hypothesis of a nonlinear term (or dummy variable) can be rejected, then linearity is rejected in favor of the alternative hypothesis of a two-regime threshold model. Following Quandt (1960) and Davies (1987), a SUP-LM test can be used to evaluate the maximum value of the computed statistics. We use a Lagrange multiplier (LM) procedure since it does not require estimation of the unrestricted model, which is sometimes difficult in nonlinear estimation. The LM statistic is computed on a grid of candidate values for the threshold for the parameter space $\pi > 0$, where $\pi = .15$ (following Andrews, and Ploberger, 1994). The sorted value that maximizes the

SUP-LM statistic is then selected as the threshold statistic. We also employ the AVE-LM statistic that is optimal against local alternatives, and use a similar grid search and LM test to evaluate the significance of this statistic. To introduce the main results, suppose that interest rate differentials were generated from a near-integrated TAR model with errors that follow a GARCH(1,1) process

$$\begin{aligned}\Delta y_t &= \mu + \rho y_{t-1} + I_{\{z_{t-d} \geq \lambda\}}(\gamma + \phi y_{t-1}) + \sqrt{h_t} \xi_t \\ h_t &= \omega + \alpha \varepsilon_{t-1}^2 + \beta h_{t-1}\end{aligned}\tag{4}$$

with $\rho = c/T, \omega > 0, \alpha \geq 0, \beta \geq 0$. It is further assumed that the standardized errors $\xi_t = \varepsilon_t / \sqrt{h_t}$ are *iid* with $E(\xi_t) = 0, E(\xi_t^2) = 1, E|\xi_t|^{2+\varepsilon} < \infty$ for some $\varepsilon > 0$ and $E \ln(\alpha \xi_t^2 + \beta) < 0$. Since the limiting distributions of the test statistics are difficult to derive, Gospodinov (2005) employs Monte Carlo simulations to gain insight into the effects of heteroscedasticity on size and power. To mimic the error structure of the data, he employs three different bootstrap methods; fixed regressor bootstrap of Hansen (2000a, 2000b), a GARCH(1,1) bootstrap, and a feasible GLS bootstrap. The fixed regressor bootstrap of Hansen (2000a, 2000b) holds all the regressors including lagged dependent variables fixed across replications and repeatedly draws from the data a large number of times. Due to its fixed structure, it can only provide a first order approximation to the finite-sample distribution of the test statistic. If we have some a priori information about the form of the heteroscedasticity, then we can model jointly the conditional mean and the conditional variance, which would presumably result in efficiency and power gains. It is a well established empirical result that the GARCH(1,1) process provides a very good approximation to the heteroscedastic error processes of many financial variables, including interest rates. Lastly, the null hypothesis is evaluated using a feasible GLS bootstrap that allows for more general forms of heteroscedasticity than the GARCH(1,1) framework.

Additional details on the bootstrap procedure can be found in appendix I. In this analysis the data possess an unknown small sample distribution which may be better approximated by a bootstrap than an asymptotic distribution.⁵

2C. Symmetric Thresholds

One limitation of both the Caner and Hansen and the Gospodinov (2005) approach is that they considers only a one-sided threshold, however a two-sided threshold may exist. In this paper, we calculate a symmetric threshold. First, we extend equation (3) to allow for a two-sided symmetric threshold by replacing λ with $|\lambda|$. The investigation of a symmetric threshold is straightforward. We take the absolute value of the nuisance parameter and sequentially substitute it into equation (3) to find the largest SupF_T statistic.

We employ a *symmetric* mean reversion TAR model and let y_t from equations (2-4) equal $(i - i^*)_t$, where i is the US short-term interest rate and i^* is the foreign short-term interest rate. We set the delay parameter equal to unity since for a financial variable it is likely to be short and designate the threshold variable to be $|z_{t-1} = \Delta(i - i^*)_{t-1}|$. To make things more explicit, note that a symmetric TAR model of this type can be written as:

$$\begin{aligned} \Delta(i - i^*)_t &= \rho_2(i - i^*)_{t-1} + \varepsilon_t \quad \text{if } \Delta(i - i^*)_{t-1} > \lambda \\ &= \rho_1(i - i^*)_{t-1} + \varepsilon_t \quad \text{if } |\Delta(i - i^*)_{t-1}| \leq \lambda \\ &= \rho_2(i - i^*)_{t-1} + \varepsilon_t \quad \text{if } \Delta(i - i^*)_{t-1} < -\lambda \end{aligned} \tag{5}$$

The degree of mean reversion within the threshold bands is given by ρ_1 and is hypothesized to be near zero (a unit root if it is insignificantly different from zero). The degree of mean reversion in the upper and lower thresholds are assumed equal and given by ρ_2 , and is hypothesized to be

mean reverting, $\rho_2 < 0$. We restrict the degree of mean reversion in the upper and lower threshold to be equal. Empirical analysis reveals that this hypothesis cannot be rejected and thus, imposing the restricting increases the power of our subsequent tests. If $\rho_1 \neq \rho_2$ the degree of mean reversion beyond the upper and lower thresholds is different than that within the threshold bands.

The threshold parameter λ is unknown and can be endogenously determined.⁶ A threshold exists when we can reject the null hypothesis that $\rho_1 = \rho_2$. When $|\Delta(i-i^*)_t| \leq \lambda$, and $\rho_1=0$ then $(i-i^*)_t$ is a random walk. When $|\Delta(i-i^*)_t| > \lambda$, and $\rho_2 < 0$ then $(i-i^*)_t$ follows a stationary mean reverting autoregressive process. Note that we use the lagged one period change in the interest rate differential as the threshold variable. This specification is appropriate in the present context because the statistical inference for threshold nonlinearity is derived under the assumption of a stationary threshold variable. Hansen (1997) presents a statistical argument for the M-TAR adjustment. He finds that if the threshold variable (i.e. the variable that governs the switch between regimes) is a near unit root process, then it is safer to work with the differenced threshold rather than its level. As the interest rate differential has been shown to be strongly persistent and close to a unit root process, we use the lagged change in the interest rate differential as the threshold variable.⁷

3. Empirical Results

The data employed in this paper are monthly short-term, money market interest rates for the US, Germany, France, and the UK obtained from the IFS database. We employ the US-UK, US-German and US-France interest rate differential. The sample period for the US, France and Germany is 1964.01-1998:08. The sample period for the UK is 1975.06-1998.08. To determine

whether both the level of the interest rate and interest rate differential for each country follow a unit root process, we employ ADF unit root tests. All interest rate differentials are expressed as the US interest rate minus the foreign interest rate. The selection of lagged difference terms in the ADF regression is chosen using a 10% top down testing procedure beginning with a maximum lag of 12 months. ADF test results are reported in Table 1.

The results indicate that a unit root process cannot be rejected for either the level of interest rates or the interest rate differential in all cases at the 5% level of significance. The results (not reported) when Germany is the benchmark also indicate that the null of unit root cannot be rejected. Enders and Granger (1998) show that tests for unit roots and cointegration all have low power in the presence of asymmetric adjustment (such as a TAR process). We next illustrate the existence of heteroscedasticity in the interest rate differentials and then investigate the existence of nonlinear threshold behavior.

Table 2 presents some preliminary diagnostic tests. The top panel investigates the residuals from an AR(1) model of the interest rate differentials. The reported p-values from an ARCH-LM test of the residuals (row 1) indicate that the null of a homoscedastic error process is strongly rejected. There is also strong evidence of kurtosis (row 3) in the residuals (as the statistic exceeds 3) in all three cases, as well as some negative skewness (row 2) with respect to the US-France AR(1) residuals and skewness for France and Germany. The interest rate differentials exhibit leptokurtic behavior in all cases. In modeling the residuals as an ARCH and GARCH process, we note that the ARCH (row 4) and GARCH (row 5) parameters are very significant, indicating the presence of GARCH effects for all interest rate differentials. After adjusting for the GARCH (1,1) effects, we again conduct the ARCH-LM test on the residuals from the AR(1) model of the interest rate differentials and report the p-values (row 6). Once the

GARCH (1,1) process of the residuals from the AR(1) model of the interest rate differentials has been modelled, we cannot reject homoscedastic residuals. In addition, the GARCH framework reduces the kurtosis of the residuals in half.

In Panel B of Table 2 we turn our attention to the interest rate differentials themselves. We compute the unconditional variance of our monthly interest rate differentials for rolling twelve month periods. In row 9 we report the highest and lowest values of these variances. The dispersion of the yearly variances over our sample period is quite large. Such dramatic changes in the unconditional variance support the existence of heteroscedasticity. We next examine the size distortions in ADF unit root tests when the error process is heteroscedastic. Following the procedure of Seo (1999), we perform a Monte Carlo simulation (3000 repetitions) study with a unit root null and an error structure given by the ARCH and GARCH parameters estimated in rows (4) and (5). We find that the critical values dramatically shift more negative yielding very large size distortions (row 10a). Standard ADF tests, such as those reported in Table 1, are not reliable in cases of highly persistent series with a heteroscedastic error processes. Gospondinov (2001) also reports substantial differences in the critical values from a unit root Monte Carlo procedure that models the GARCH process.

We next conduct endogenous structural change tests to determine whether a break exists in the estimated parameters or variance structure using the methodology of Hansen (2000b). A structural break can also lead to false inference in unit root tests including TAR methods, and hence we would like to examine whether a permanent change exists. Rows 11a, 11b, 11c, and 11c, report the test statistic, the Andrews (1993) asymptotic p-value, the homoscedastic p-value, and the heteroscedastic p-value. For all three interest rate differentials, the Andrews p-values associated with the SUP-F tests indicate that the null of stability against the alternative of a

single structural break can be rejected, while the AVE-F tests indicates that the null of stability against the alternative of a single structural break cannot be rejected. The potentially more reliable homoscedastic bootstrapped p-values indicate that for the US-France and US-German interest rate differentials, stability cannot be rejected. When these test statistics are adjusted to account for heteroscedasticity (row 11d) we find that both the SUP-F and the AVE-F tests indicate stability cannot be rejected. Such evidence indicates that ignoring heteroscedasticity may lead to misleading inferences with respect to a single structural break. The substantial differences between p-values for the homoskedastic and heteroskedastic bootstraps is consistent with both Hansen (2000b) and Strauss and Yigit (2001) who report four to six decimal place difference for the SupLM p-values for some series.

We begin our investigation of threshold behavior in interest rate differentials by examining whether the interest rate differential can be modeled as a symmetric threshold model with interest rate differential exhibiting (near) unit root behavior inside the band and exhibiting mean reverting behavior outside the band. We employ the methodology of Gospodinov (2005) and allow for a GARCH(1,1) error process and test for the existence of thresholds. To allow for a symmetric threshold we employ the absolute value of the lagged change in interest rate differential as the threshold variable (absolute value of z_t in equations 2-4 above). This yields a symmetric band around zero. We employ both the SUP-LM and the AVE-LM tests for the existence of a threshold. These procedures are designed to test the null hypothesis that the intercepts and autoregressive coefficients are the same in both regimes versus an alternative of a symmetric TAR model.

Before discussing the findings of our LM tests, it is worth noting the similarities between endogenous structural change SUP and AVE tests and threshold SUP and AVE tests. The AVE-

LM and SUP-LM tests are derived from the optimality conditions of Andrews (1993) and Andrews and Ploberger (1994), and possess the same nonstandard asymptotic distributions found by Hansen (1992). Hansen (1992) shows that parameter stability tests can evaluate the existence of a cointegrating vector, and since the alternative hypothesis of a random walk in the intercept is identical to no cointegration, the test statistics are tests of the null of cointegration against the alternative of no cointegration. The average (AVE) LM and supremum (SUP) LM employed in Hansen (1992) are LM statistics for structural change in the cointegrating vector with an unknown breakpoint. These procedures have been amended to test for a threshold. Hansen (1997, 2000a) shows that the testing procedures are similar - both the endogenous structural break and the TAR procedure use nuisance parameters and the SUP or AVE LM statistics as test criterion for evaluating/determining the parameter that minimizes the error in the estimated equations. The AVE-LM statistic tests whether the specified model is a good model that captures a stable relationship. A large test statistic implies that there is a break or threshold. The SUP-LM is appropriate for testing a swift shift in regime and its power is also concentrated on the distant alternative hypothesis. We use this SUP-LM statistic to evaluate differences in the error between models. A large test statistic implies that we can reject a linear specification.

We now turn to a discussion of our LM test results. Rows 1-3 in Table 3 report the homoskedastic fixed regressor (HOM-LM), a feasible GLS (HET-LM) and the GARCH(1,1) (GARCH-LM) bootstrap test statistics. As we are concerned with the results based on a GARCH(1,1) error process, we focus most of our attention on the GARCH-LM test. While the HOM-LM (row 1) and HET-LM (row 2) tests give mixed results, the GARCH-(SUP) LM (row 3) test rejects the null of no threshold in favor of a symmetric threshold for all three interest rate differentials. The GARCH (AVE)-LM test rejects one regime in favor of a symmetric threshold

for the US-German interest rate differential, at the 5% level, and for the US-UK interest rate differential at the 10% level. The results of the LM tests indicate that allowing for a heteroscedastic error process can be important in detecting a threshold. Assuming a homoscedastic process may lead one to falsely infer that no threshold exists. We note that the use of alternative trimming parameters [.05 .95 or .10 .90] in Table 6 (discussed later) yield even more significant evidence of a threshold.

Row 5 and 6 of Table 3 report the value of ρ_1 (corresponding to the measure of persistence inside the symmetric threshold bands) and ρ_2 (corresponding to the measure of persistence outside the threshold bands). We report both the standard errors and bootstrap t-statistics using our GARCH (1,1) framework. A value that is insignificantly different from zero indicates that a unit root process cannot be rejected. For conciseness, we do not report the intercept terms. For the US-France and US-German interest rate differentials, we find that the interest rate differential is mean reverting outside the threshold bands as the unit root null can be rejected (row 6). The coefficient ρ_1 (associated with behavior inside the threshold bands) follows a unit root (mean-reverting) process for the US-German (US-France) interest rate differential. Hence, a partial unit root process exists for the US-German interest rate differential, while the US-France interest rate differential can be characterized by a stationary TAR-GARCH(1,1) model. Although we cannot statistically argue that the US-France interest rate differential is a unit root inside the threshold bands, the magnitude of ρ_1 (inside the bands) is much smaller than ρ_2 (outside the bands). For the US-UK interest rate differential we cannot reject unit root behavior in either of the two regimes, however, the magnitude of ρ_1 (within the bands) is smaller than that of ρ_2 (outside the bands). For all three interest rate differentials, the evidence suggests that the interest rate differentials are more persistent within the threshold

bands than outside the bands. The total percentage of observations outside the threshold bands is reported in row 4 of Table 3. For the US-German interest rate differential 15% of the observations lie outside the threshold bands, while 30% lie outside the threshold bands for the US-France interest rate differential. For the US-UK interest rate differential 45% lie outside the threshold bands. The threshold values are reported in row 7.⁸ As an illustrative example, we find that when the lagged change in the interest rate differential is above 0.47 or below -0.47 , the interest rate differential is mean reverting, as our bootstrap critical values indicate that we can reject the unit root null (row 6). However, our bootstrapped critical values indicate that inside these bands, the interest rate differential exhibits unit root behavior (row 5). Rows 8 and 9 of Table 4 report the ARCH and GARCH parameters that are all significantly different from zero.

Figure 1 provides a plot of *change* in the interest rate differential for US-Germany (Panel A) and US-France (Panel B) and the estimated threshold bands. With respect to the US-Germany linkage, it is clear from the short-lived large (but narrow) spikes, that when the change in the interest rate differential in the previous period is large there is strong mean reversion. However, when the change in the interest rate differential is small and within the bands, the series are much more persistent. The large spikes appearing around 1973 are most likely the result of the movement away from the Bretton Woods System. Large spikes are also seen in the very late 1970s and early 1980s. While no spikes appear in the figure for US-France during the mid-1970s, they do appear around 1980.

We next investigate the forecasting ability of the TAR model described in Table 3. In Table 4 we report the Root Mean Squared Errors (RMSE) at horizons of 3, 6, 9, 12, and 18 months for the interest rate differential modeled as a random walk (RW), an autoregressive model of order one, AR(1), and a TAR-GARCH(1,1) model. For the both the US-German

interest rate differential and the US-France interest rate differential, with the exception of the 3-month horizon, the TAR-GARCH(1,1) model outperforms both the RW and the AR(1) model at all horizons. The US-UK interest rate differential model does not fair as well. With respect to the US-UK interest rate differential, the TAR-GARCH(1,1) model outperforms the RW model at the 6,9 and 12 month horizons. At the 3 and 18 month horizon, the RW model outperforms both the AR(1) and the TAR-GARCH(1,1).

Table 5 alters the threshold variable and considers the absolute value of the lagged change in the level of the short-term domestic or foreign interest rate. Previous research has reported that short-term interest rate can be effectively modeled as a nonlinear process. If innovations in the domestic rate are not matched contemporaneously by identical movements in the foreign rate, the interest differential will change. Hence, we hope to gain some insight as to which variable triggers the nonlinear innovations in the interest rate differential, and explains movements from one regime to another. The results provide some interesting insights. Recall that the strongest evidence in favor of a TAR-GARCH(1,1) process is found for the US-German interest rate differential and the US-France interest rate differentials. For the results in Table 5, we find that (with the exception of the GARCH-AVE) all three of the LM tests reject the null of one regime for the US-German and US-France interest rate differential when the threshold is the absolute value of the change in the lagged short-term US interest rate. For these cases, when the absolute value of the lagged change in the foreign interest rate is the threshold variable, at least one of the SUP or AVE tests for both the HOM-LM and HET-LM statistics rejects the null of one regime in favor of a symmetric TAR process. For the US-UK interest rate differential, regardless of whether the absolute value of the lagged domestic or foreign rate is the threshold variable, both the SUP and AVE statistics for the GARCH-LM test reject the null of one regime.

We now turn to the degree of persistence within each regime. With the exception of the US-German interest rate differential (when the absolute value of the lagged change in the US short-term rate is the threshold) we find that ρ_2 (degree of mean reversion outside the bands) indicates that the null of unit root can be rejected. We find that ρ_1 (degree of persistence inside the threshold bands) is insignificantly different from zero indicating unit root behavior. This suggests that a partial unit root TAR model likely characterizes interest rate differentials. For the cases in which the absolute value of the lagged short-term US interest rate is the threshold variable, we see that the value of ρ_2 for each interest rate differential, are similar in size to those found in Table 3 (where we did not separate out the changes in the short-rates as threshold variables). When the absolute value of the lagged change in the foreign interest rate is the threshold (with the exception of the UK-US interest rate differential) the value of ρ_2 is much smaller in absolute value than the values reported in Table 3. It is also interesting to note that for the US-German and US-France interest rate differential, the value of ρ_2 is larger, in absolute value, relative to the value of ρ_2 when the absolute value of the lagged change in the foreign short-term rate is chosen as the threshold. These observations suggest that it may be the change in the US short-term interest rate that is providing the trigger for switching from one regime to another relative to the effects of changes in the foreign interest rate. While we did not find evidence of a TAR process for the US-UK interest rate differential in Table 3, we do find evidence of a partial unit root TAR process in Table 5.

In the estimation of TAR models a trimming parameter must be specified (see footnote 5). The choice of the trimming parameter π_i in practice is guided by the consideration that each regime must have a sufficient number of observations to adequately identify the regression parameters. In our analysis up to this point we followed Andrews (1993) recommendations for

the SUP LM and selected a trimming parameter interval of [.15 .85]. In order to examine how robust our results are to the choice of these trimming parameters we repeat the analysis of Table 3 for trimming parameters [0.05 .95] and [.10 .90]. These results are reported in Table 6. The results are qualitatively the same as in our previous analysis of Table 3.

In our final analysis, we follow Gospodinov (2005), Stanton (1997) and Ang and Bekaert (1998) and employ nonparametric methods to estimate the drift and the diffusion (or volatility) function of the underlying process. The drift and the diffusion function that are implied by the TAR-GARCH(1,1) model are calculated from a long realization of the process (200,000 observations) obtained recursively by drawing from the empirical distribution function of the residuals and the estimated parameters given in Table 3. The nonparametric estimates of the drift and the diffusion functions are plotted in Figure 2 for the US-German interest rate differential (top panel) and for the US-UK interest rate differential (lower panel). The top panel of Figure 2 illustrates that for the absolute value of the US-German interest rate differential, the drift is positive at low levels (and relatively flat) and negative (and downward sloping) at high levels. The drift function suggests that the process is mean reverting at very high levels of the (absolute value) interest rate differential. This drift in the model resembles that found in Stanton (1997), Ang and Bekaert (1998) and Gospodinov (2005) for the level of the short-term interest rate. The drift is flat at low levels of interest rate differentials and then it turns negative at higher levels. In the middle range, the series behaves like a random walk. Further, a graph of the drift and diffusion process of the short term interest rate (not reported) using a TAR GARCH (1,1) mimics the graph of the interest rate differentials closely, indicating that the nonlinear process of the differentials is likely originating from innovations in the U.S. short term interest rate.

The conditional volatility function (in the right panel) has the same J-shape that Stanton

(1997) and Gospodinov (2005) found for the level of the short-term interest rate. This shows that volatility is high at high values of the absolute value of the interest rate differential (potentially outside the threshold bands) but at low levels of the absolute value of the interest rate differential (potentially inside the band) the volatility is low. The lower panel of Figure 2 plots similar drift and volatility functions for the US-UK interest rate differential. While the drift function is similar to the US-German interest rate differential (although not quite as pronounced), the volatility function has more of a U shape rather than the J-shape. This is consistent with our results in Table 3 where we find strong evidence for a symmetric partial unit root TAR model for the US-German interest rate differential but weaker evidence for a TAR process for the US-UK interest rate differential. We, thus find that the GARCH (1,1) TAR framework is able to model the non-parametric estimate of the drift and volatility of the interest rate differential very well. The TAR model generates similar drift and volatility behavior found in other papers for the level of the short-term interest rate.

4. Conclusions

The failure of uncovered interest parity continues to puzzle economists despite a burgeoning literature. Uncovered interest parity argues that the percentage change in the expected exchange rate should equal the interest differential (plus a risk premium). The first component possesses little serial correlation and is rapidly mean reverting, while the interest differential, according to ADF tests, is very persistent. One explanation for the apparent unit root behavior is that the interest rate differential follows a nonlinear TAR process, where small deviations are persistent, but too small to be arbitrated away because of uncertainty and/or transactions costs. Larger deviations, on the other hand, that exceed a threshold are arbitrated

away, and are mean reverting. Hence, a band or threshold may exist around interest differentials, and the degree of mean-reversion depends on the size of the previous change in the interest rate differential.

In this paper we examine interest rate differentials between the US and Germany, US and France, and US and the UK, and our procedure models the heteroscedasticity in the data. We test for a partial unit root process – the existence of a central bank of “inaction” where the interest rate differential follows a persistent or (near) unit root process for small innovations within the threshold bands, while exhibiting mean reversion behavior when outside the symmetric bands. Using one-month interest rate differential data, we significantly reject linearity in favor of the alternative hypothesis of a two-regime threshold model.

Our results support the partial unit root TAR process for the US-German interest rate differential, and are robust to whether the threshold is the absolute value of the lagged change in the interest rate differential or interest rate. We find evidence of a stationary TAR process for the US-France interest rate differential when the absolute value of the lagged change in the differential is used as the threshold variable, but find evidence of a partial unit root TAR process when the absolute value of the lagged change in the short-term interest rate is used as the threshold variable. Our analysis also suggests that changes in the level of the US interest rate appear to dominate as the trigger mechanism in shifting between thresholds. Furthermore, we find that the TAR GARCH (1,1) framework models the non-parametric estimate of the drift and volatility of the interest rate differential very well. The TAR modeling framework can replicate results for the drift and volatility that other papers have reported for the level of short-term interest rates, and suggest that that the origin of the innovations in the interest rate differential are the nonsimultaneous movements in the domestic or foreign interest rate.

Appendix I TAR with GARCH Errors

Since many authors have shown that the GARCH(1,1) is a good approximation to the error heteroscedasticity of many financial variables, one can increase efficiency and power by jointly modeling the conditional mean and variance as a GARCH(1,1). To obtain the finite sample critical values, we first estimate using quasi ML the threshold assuming a GARCH(1,1) To introduce the main results, suppose that interest rate data were generated from the near-integrated TAR with errors that satisfy Assumption 1 and follow a GARCH(1,1) process

$$\Delta y_t = \mu + \rho y_{t-1} + I_{\{z_{t-d} \geq r\}}(\gamma + \phi y_{t-1}) + \sqrt{h_t} \xi_t = \omega + \alpha \xi_{t-1}^2 + \beta h_{t-1} \quad (\text{A.1})$$

with $\rho = c/T, \omega > 0, \alpha \geq 0, \beta \geq 0$. It is further assumed that the standardized errors $\xi_t = \varepsilon_t / \sqrt{h_t}$ are *iid* with $E(\xi_t) = 0, E(\xi_t^{2+\varepsilon}) < \infty$ for some $\varepsilon > 0$ and $E \ln(\alpha \xi_t^2 + \beta) < 0$. Following Caner and Hansen (2001) and Gospodinov (2005), the threshold is stationary and an ergodic random variable.

The asymptotic representations of the standardized quantities $T^{-1/2} \sum_{t=1}^T A_t^1, T^{-1} \sum_{t=1}^T A_t^2, T^{-1/2} \sum_{t=1}^T B_t^1$ and $T^{-1} \sum_{t=1}^T B_t^2$, are difficult to derive due to the nonlinear estimation of the GARCH models. As for the heteroscedasticity-robust LM test, we only conjecture that under the null of no threshold effect in model (3), the distribution of the $SupF_T$ test for linearity can be reasonably approximated by a limiting distribution obtained under the assumption of conditionally homoscedastic errors.

Alternatively, conditional on the data, we can use bootstrap methods to approximate the finite sample critical or p -values of the test statistic. Assume that there exists a limiting distribution $\sup F$ such that $\sup F_T$ converges weakly to $\sup F$ as $T \rightarrow \infty$. Then, the bootstrap p -value of the $SupF_T$ test is approximated through the following procedure. First, estimate the GARCH(1,1) by quasi ML and compute the test statistic $SupF_T$. Calculate the standardized residuals under the null $\hat{\xi}_t = \hat{v}_t / \sqrt{\hat{h}_t}$. Since ξ_t are assumed to be *iid*, then we can resample with replacement directly from their empirical distribution function to obtain the sequence $\{\xi_t^*\}$. Then, for some initial conditions h_0^*, ξ_0^* and y_0^* , the bootstrap series $\{y_t^*\}$ is constructed recursively under the null from

$$h_t^* = \hat{\omega} + (\hat{\beta} + \hat{\alpha}\xi_{t-1}^{*2})h_{t-1}^*$$

$$y_t^* = \hat{\mu} + (1 + \hat{\rho})y_{t-1}^* + \sqrt{h_t^*}\xi_t^* .$$

This algorithm is repeated B times and each time the statistic $SupF_T^*$ is computed. Then, the p-value of the test is given by the probability $\Pr\{SupF_T^* \geq SupF_T \mid \theta = \hat{\theta}\}$. Lastly, we evaluate the null hypothesis using a feasible GSL bootstrap that allows for more general forms of heteroscedasticity than the GARCH(1,1). See Gospodinov (2005; p. 12) for more details.

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Table 1: ADF Unit Root Tests

Country	UK	FRA	GER	US
i^* (Level)	-2.25	-2.44	-2.27	-1.51
α	0.955	0.979	0.954	-0.982
Lag length	8	6	9	11
$i^{us} - i^*$	-2.51	-1.44	-2.34	
α	0.912	0.978	0.952	
Lag length	11	6	8	

The 5% Critical value is -2.87. The coefficient α is the sum of the autoregressive coefficients. Sample period for US, Germany, and France: 1964:01-1998:08. Sample period for the UK: 1975:06-1998:08.

Table 2: Preliminary Diagnostic Tests

Interest Rate Differential	$i^{\text{US}} - i^{\text{UK}}$	$i^{\text{US}} - i^{\text{Fra}}$	$i^{\text{US}} - i^{\text{Ger}}$
Tests of Residuals from an AR(1) process of the interest rate differentials			
1. ARCH-LM (p-value)	[0.000]	[.006]	[0.000]
2. Skewness	-0.01	-1.527	1.6434
3. Kurtosis	10.68	16.62	7.8522
4. ARCH parameter (S.E.)	0.16** (0.05)	0.3342** 0.1253	0.6011** 0.0759
5. GARCH parameter (S.E.)	0.83** (0.03)	0.6608** 0.07309	0.3939** 0.0600
6. After adjusting for GARCH Effects ARCH-LM P-values	[0.85]	[0.3015]	[0.3105]
7. Skewness after adjusting for GARCH effects	-0.06	1.131	0.7951
8. Kurtosis after adjusting for GARCH effects	5.09	7.621	4.3347
Panel B: Analysis on the Interest Differential themselves			
9. Variance: Hi	16.02888	59.35475	16.93471
Low	0.021336	0.012475	0.012408
10. Size distortion and critical values	5% 10%	5% 10%	5% 10%
10a. Size of ADF tests when hetero. is present	.412 .543	0.452 .592	0.781 0.847
11. Structural Break Tests	SUP F AVE F	SUP F AVE F	SUP F AVE F
11a. Test Statistic	35.636 5.336	16.425 4.127	21.079 3.640
11b. Andrews P-values	[0.000] [0.940]	[0.047] [0.880]	[0.008] [0.494]
11c. Homoscedastic Bootstrap P-values	[0.000] [0.153]	[0.122] [0.465]	[0.060] [0.267]
11d. Heteroscedastic Bootstrap P-values.	[0.136] [0.960]	[0.227] [0.397]	[0.159] [0.422]

** Significant at 1%, *significant at 5%, †Significant at 10%. P-values in brackets

LM p-values. H_0 is no conditional heteroscedasticity. Rows 4 and 5 report ARCH and GARCH parameter estimates with QMLE robust standard errors. ARCH p -values for GARCH(1,1).

Table 3 Interest Differential Adjusting for Heteroscedasticity: Symmetric Thresholds

$$\Delta y_t = \mu + \rho y_{t-1} + I_{\{z_{t-1} \geq \lambda\}} (\gamma + \phi y_{t-1}) + \varepsilon_t, t = 1, \dots, T,$$

where $y = (i - i^*)$ and $|z| = \Delta(i - i^*)$

Interest Rate Differentials	$Y=i^{US}-i^{UK}$	$y=i^{US}-i^{Fra}$	$Y=i^{US}-i^{Ger}$
LM tests for Symmetric Threshold			
1. HOM Sup	2.115	15.27	20.29*
Ave	0.6046	10.89*	10.29
2. HET Sup	1.363	16.42**	6.06
Ave	0.4881	10.4**	3.46
3. GAR Sup	9.52*	15.217**	35.42**
Ave	3.67 ⁺	2.288	8.29**
4. % of obs. in outer regimes	45	30	15
5. ρ_1 (inside the bands)	-0.073	-0.037*	-.028
(S.E.)	(0.050)	(0.014)	0.01
P-value	[0.341]	[0.037]	[0.059]
6. ρ_2 (outside the bands)	-0.126	-0.214**	-0.076**
(S.E.)	(.039)	(0.074)	(0.031)
P-value	.225	[0.00]	[0.002]
7. Threshold Value λ	0.11	0.2100	0.4700
8. ARCH parameter	0.164**	0.451**	0.247**
(S.E.)	(.05)	(0.15)	(0.11)
9. GARCH parameter	0.831**	0.544**	0.747**
(S.E.)	(.04)	(0.09)	(0.072)

** Significant at 1%, *significant at 5%, ⁺Significant at 10%. LM p-values. H_0 is no conditional heteroscedasticity. ARCH p values of GARCH(1,1). HOM is the fixed regressor bootstrap under homoscedasticity, HET uses a bootstrap that allows for a general form of heteroscedasticity (see Hansen 2000b). HET employs a bootstrap under a GARCH(1,1) specification of the residuals (see Gospodinov 2005). SupLM and AveLM statistics reported.

Table 4: RMSE of the k-step forecast

	$i^{US}-i^{UK}$			$i^{US}-i^{Fra}$			$i^{US}-i^{Ger}$		
RMSE Months	RW	AR(1)	TAR	RW	AR(1)	TAR	RW	AR(1)	TAR
3	0.1304	0.1562	0.1636	0.8428	0.828	0.8559	0.487	0.485	0.496
6	0.313	0.5674	0.1863	2.612	2.333	2.191	1.530	1.425	1.3352
9	0.6724	0.6087	0.6523	3.006	2.371	2.241	1.731	1.558	1.409
12	0.8056	0.8363	0.7721	3.06	2.097	2.062	1.793	1.592	1.466
18	0.5182	1.072	0.7449	3.708	2.222	2.15	1.934	1.678	1.486

Table 5: Interest Differential Adjusting for Heteroscedasticity: Symmetric Threshold
Threshold variable: Change in Short-rate

$$\Delta y_t = \mu + \rho y_{t-1} + I_{\{z_{t-1} \geq \lambda\}} (\gamma + \phi y_{t-1}) + \varepsilon_t, t = 1, \dots, T,$$

Interest Rate Differential Threshold variable	$y = i^{US} - i^{UK}$ $z = \Delta i^{UK} $	$y = i^{US} - i^{UK}$ $z = \Delta i^{US} $	$y = i^{US} - i^{Fra}$ $z = \Delta i^{Fra} $	$y = i^{US} - i^{Fra}$ $z = \Delta i^{US} $	$y = i^{US} - i^{Ger}$ $z = \Delta i^{Ger} $	$y = i^{US} - i^{Ger}$ $z = \Delta i^{US} $
1. LM test for Threshold						
2. HOM Sup Ave	6.363 2.533	15.96 7.618	39.38** 16.10**	62.95** 47.51**	21.76* 6.76	33.93** 23.69**
3. HET Sup Ave	7.913 2.720	7.024 4.164	10.82 6.42*	67.65** 44.81**	9.53* 3.89	51.73** 33.85**
4. GAR Sup Ave	28.30** 9.802**	23.97** 8.811**	5.52 1.89	10.38* 3.74	8.41 3.61	16.02** 3.30
5. % of obs. in Regime 2	35	22	63	15	60	31
6. ρ_1 (inside bands) (S.E.) P-value	-.082 0.044 .277	-.054 0.034 .214	.01 (.02) .385	-.01 (.01) .590	0.00 (.018) .128	-.01* (.007) .038
7. ρ_2 (Outside bands) (S.E.) P-value	-0.135* (.028) .028	-.129* (0.053) .021	-.029* (.012) .006	-.129** (.022) .000	-.025** (.006) .000	-.061** (.017) .000
8. Threshold value λ	.25	.50	.11	.72	.23	.29
9. ARCH parameter (S.E.)	0.143** 0.041	0.1385* 0.042	.426** (.174)	.542** (.19)	.379* (.14)	.380* (.15)
10. GARCH parameter (S.E.)	0.8514 0.02964	0.852** 0.043	.570** (.160)	.449** (.145)	.616** (.080)	.615** (.090)

Table 6 Symmetric Threshold using alternative trimming parameters

$$\Delta y_t = \mu + \rho y_{t-1} + I_{\{z_{t-1} \geq \lambda\}} (\gamma + \phi y_{t-1}) + \varepsilon_t, t = 1, \dots, T,$$

where $y = (i - i^*)$ and $|z| = \Delta(i - i^*)$

Upper and Lower Trimming Parameters	UK		UK		FRA		FRA		GER		GER	
	.05	.95	.10	.90	.05	.95	.10	.90	.05	.95	.10	.90
LM test for Threshold												
1. HOM Sup Ave	4.28 .68	9.88 .96	30.44* 14.00*	36.68* 18.77*	36.68** 18.77**	36.42** 11.32						
2. HET Sup Ave	2.68 .61	4.16 .76	54.64** 27.78**	8.01* 3.45	26.74** 14.21**	5.72 3.26**						
3. GAR Sup Ave	33.23** 17.47**	33.23** 18.23**	39.38** 22.081**	24.66** 7.91**	26.85** 10.28**	35.32** 10.39**						
4. % of obs in Regime 2	39	37	37	52	47	29						
5. ρ_1 (inside bands) (S.E.) P-value	-.080 (.023) .947	-.08 (.023) .890	-.028 (.028) .087	-.080 (.02) .581	-.027* (.01) .025	-.025 (.03) .214						
6. ρ_2 (Outside bands) (S.E.) P-value	-.111 (.059) .907	-.122 (.06) .55	-.077** (.025) .000	-.102** (.171) .151	-.031** (.08) .009	-.051* (.008) .020						
7. Threshold value λ	.59	.54	.17	.55	0.50	.56						
8. ARCH parameter (S.E.)	.1421** (.041)	.16** (.044)	.281 (.161)	.400** (.18)	.400** (.18)	.424** (.18)						
10. GARCH parameter (S.E.)	.856** (.031)	.833** (.031)	.682** (.103)	.596** (.083)	.598** (.082)	.568** (.082)						

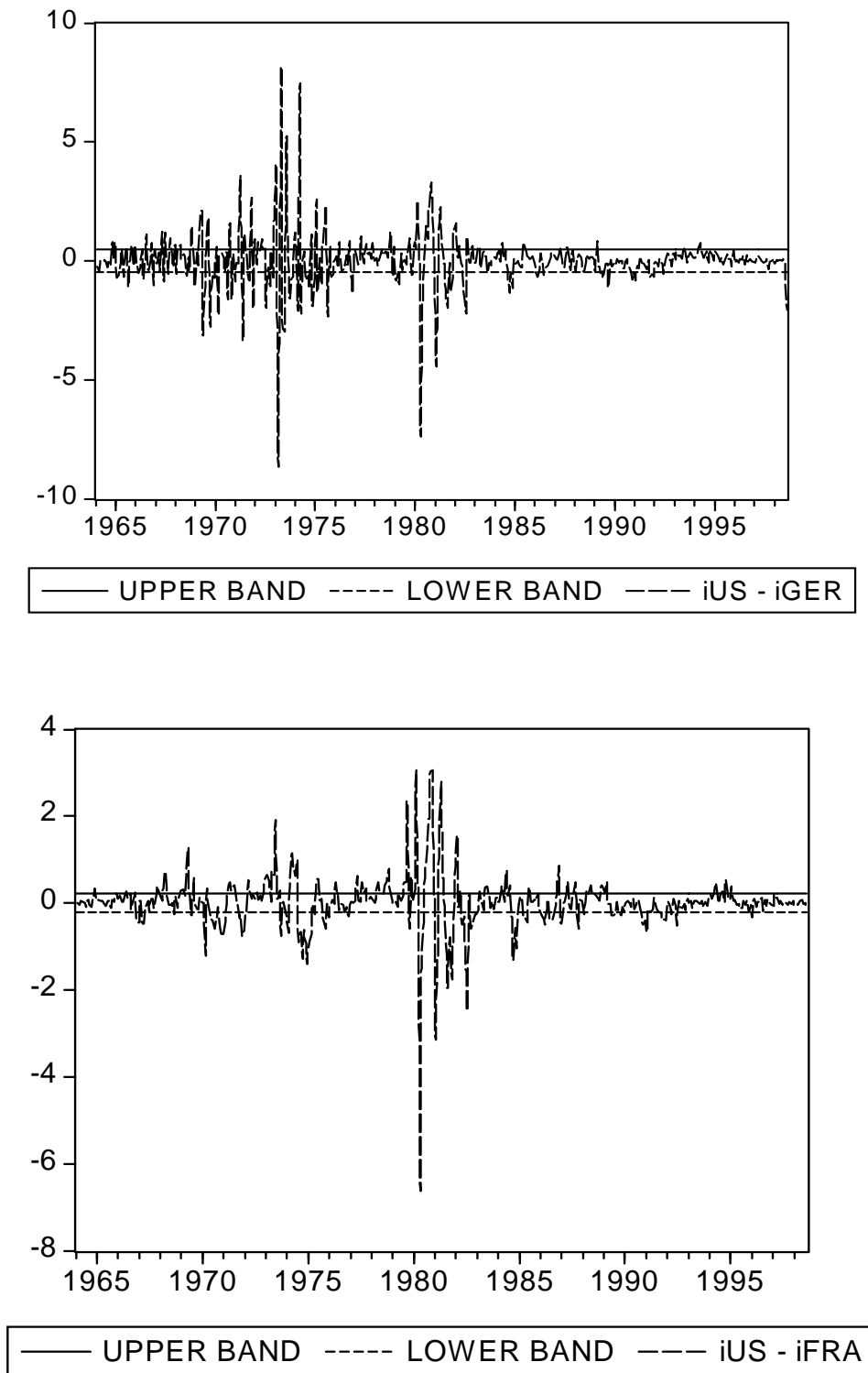
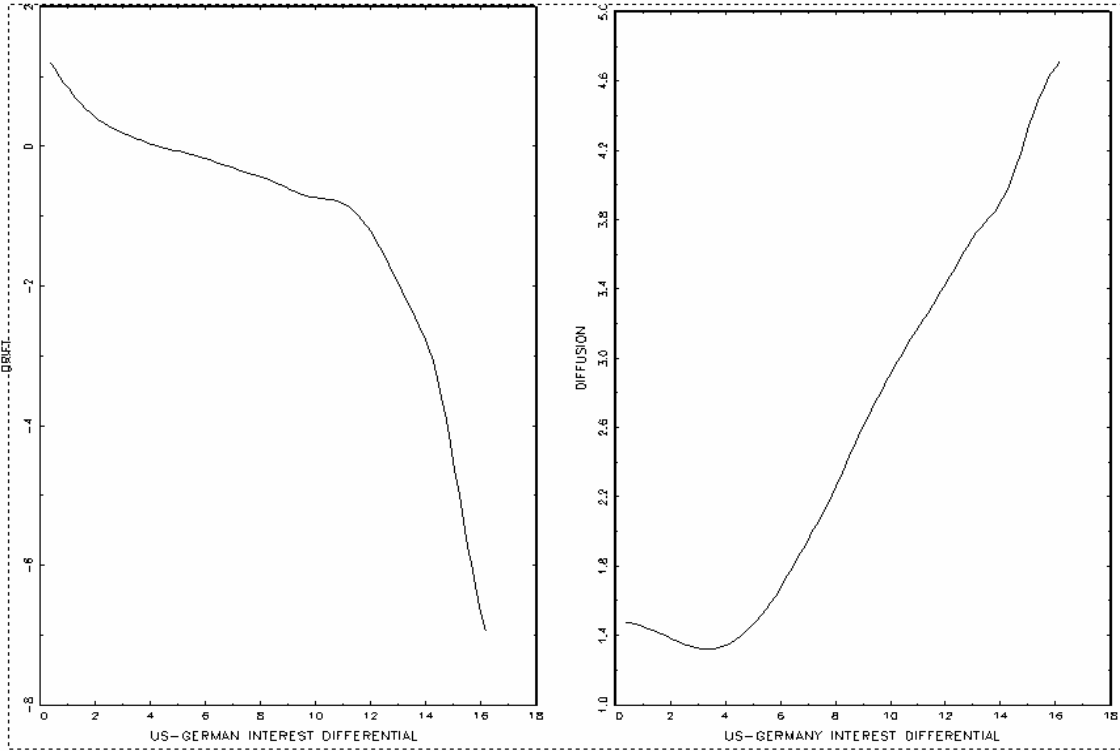
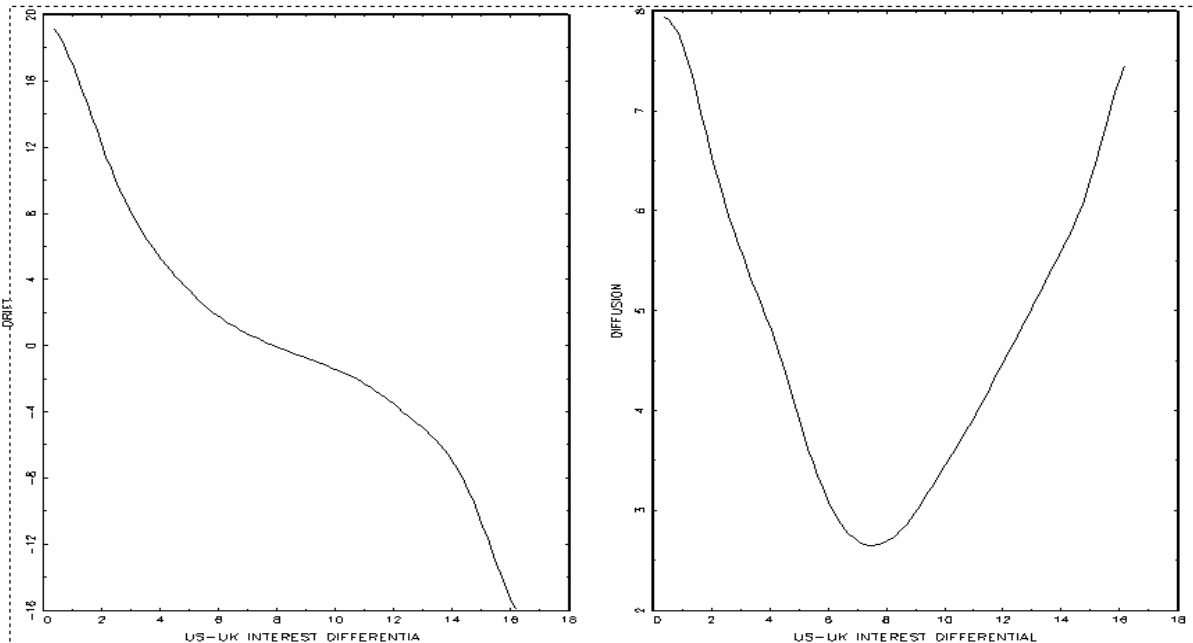
Figure 1: Plot of Change in Interest Rate Differential with Symmetric Bands

FIGURE 2:
Nonparametric Estimates of the drift (left panel) and diffusion function (right panel)
For the absolute value of the interest rate differential simulated from the
Estimated TAR-GARCH(1,1) Model with Symmetric Thresholds

US-German Interest Rate Differential



US-UK Interest Rate Differential



ENDNOTES

¹ In modeling real exchange rates, various RS models have been employed; Obstfeld and Taylor (1997) and A. Taylor (2001) employ a Band-TAR model. Michael, Nobay, and Peel (1997) and Taylor, Peel, and Sarno (2001) consider an exponential smooth transition autoregressive (ESTAR) model. Both of these models are characterized by symmetric adjustment. Bergman and Hansson (2000) estimate a two-state Markov-switching AR model of real exchange rates.

² Cecchetti, Lam and Mark (1993) and Garcia (1998) show that single-regime models are econometrically rejected in favor of their regime-switching counterpart. There are a number of studies that document nonlinear behavior in interest rates, including, Pflann, Schotman and Tscherning (1996), Ait-Sahalia (1996), Conley, Hansen and Luttmer, and Scheinkman (1997), Boudoukh, Richardson, Stanton, and Whitelaw (1997), Stanton (1997), Ahn and Gao (2000), Gospondinov (2005), and Ang and Bekaert (2002a, 2002b), among others.

³ Tong (1990) provides a survey of the statistical and dynamic properties of the TAR process. Hansen (1997, 1999) provides a review of some of the recent advances in making inferences in TAR models.

⁴ The above studies offer a number of examples to reinforce the investigation of nonlinear specifications. Gospondinov (2005) and Ang and Bekaert (2002a,b) show that interest rates can exhibit significant nonlinear behavior that depend on term structure, business cycle phenomena or the volatility ratio of long and short-term interest rates. As mentioned, we show that the nonlinear behavior of the interest rate also manifests itself in the nonlinear behavior of the interest rate differential; that is, the macroeconomic phenomena responsible for regime switching in the level of interest rate do not occur simultaneous in other economies and hence the interest rate level and differential possess similar drift and diffusion processes –at high levels, both variables are mean reverting and possess higher volatility.

⁵ A useful survey of the relevance and importance of heteroscedasticity is McAleer and Oxley, (2002), while Ruiz and Pasual (2002) surveys the application of bootstrap methods to financial time series. Davidson and MacKinnon (1999) provides a theoretical framework for the bootstrap. Li and Xiao (2001) shows that the bootstrap provides reasonably good approximation to the distribution of integrated series.

⁶ The threshold takes on values in the interval $\lambda \in \Lambda = [\lambda, -\lambda]$ where λ and $-\lambda$ are picked so that $P(Z_t \leq \lambda) = \pi_1 > 0$ and $P(Z_t \leq -\lambda) = \pi_2 < 1$. Generally, π_1 and π_2 are treated as symmetrical so that $\pi_2 = 1 - \pi_1$. This imposes the restriction that no regime has less than π_1 % of the total sample. The choice of π_1 in practice is guided by the consideration that each regime must have a sufficient number of observations to adequately identify the regression parameters. We follow Andrews (1993) and select $\pi_1 = 0.15$ and $\pi_2 = 0.85$. We also conduct analysis for $\pi_1 = 0.05$ and $\pi_2 = 0.95$, and $\pi_1 = 0.10$ and $\pi_2 = 0.90$. See Table 6. The results are not qualitatively different.

⁷ Enders and Granger (1998) and Enders and Siklos (2001) show that this specification is especially relevant when the adjustment is such that the series exhibit more momentum in one direction than the other.

⁸ The significance levels for the test statistics, HOM-LM, HET-LM, and GAR-LM, as well as the p-values associated with ρ_1 and ρ_2 in Tables 3, 5, and 6, are computed employing a bootstrap procedure discussed in Hansen (2000b) for the HOM-LM and HET-LM tests and in Gospodinov (2005) for the GAR-LM test statistic. We also provide a brief description of the bootstrap in Appendix I.