

Domestic-Foreign Interest Rate Differentials:
Near Unit Roots and Symmetric Threshold Models

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Abstract

This paper investigates the near unit root behavior of interest rate differentials across countries using a symmetric-Band-TAR model that allows for a heteroscedastic error process. We find that the time series properties of monthly short-term interest differentials over the period 1974-2005 between the U.S. and Canada, France, Germany, Japan, and the UK can be characterized by a symmetric-Band-TAR process, which can explain its (near) unit root behavior reported in the extant literature. Results significantly reject a linear model in favor of the alternative hypothesis of a two-regime symmetric threshold model that exhibits significantly greater persistence within the threshold bands than when outside the threshold bands.

1. Introduction

The spread between domestic and foreign interest rates is an important variable that central banks consider in their policies at the macroeconomic level as well as a variable of interest for investors in the foreign exchange market who are engaged in currency carry trade. A thorough understanding of the time series properties of interest rate differentials and their persistence across countries is hence of importance for both policymakers as well as investors.

It has been documented that the time series behavior of interest rate differentials across countries is characterized by high persistence and heteroscedasticity when the data frequencies are monthly or higher. Studies such as Crowder (1995) find that conventional unit root tests possess low power in discerning whether the interest differential across economies follows a stationary or unit root process, particularly when the underlying process might be subject to nonlinearities. If interest rates possess nonlinearities that differ across economies, and transactions costs imply a band of arbitrage inaction, as suggested by Anderson (1997) and others (allowing these nonlinearities to persist), then *interest rate differentials* may follow a threshold process whereby the speed with which domestic and foreign interest rates revert to some value depends on the spread between domestic and foreign interest rates. In addition, changes in business cycle conditions and monetary policy may cause real interest rates and expected inflation to behave differently during different time periods. If business cycles and monetary policy responses are nonsynchronous, the interest differential and its persistence will also be affected. In this paper we show the potential for nonlinear models, in particular, the threshold autoregressive (TAR) model that allow for heteroscedastic errors is better able to capture some important regularities in the cross-country interest rate differentials than can linear models. While there are certainly numerous other nonlinear models one might choose to

estimate, we selected to estimate a symmetric Band-TAR model based on the financial and economic underpinnings of the foreign exchange market, which are discussed in more detail in the literature below.

The TAR framework assumes that the time series properties of interest rate differentials between economies differ depending on the level of the interest rate differential. When cross country interest rate differentials exceed an estimated threshold band, the differential will exhibit a stationary mean reverting behavior towards the band, while wandering as a non-stationary random walk when the interest rate differential lies within the threshold bands. The results of our paper show that the time series properties of cross-country interest differentials exhibit significant TAR nonlinearities that can characterize their (near) unit root behavior reported in the extant literature. The methodology used in the current paper applies and extends the framework of Gospodinov (2001, 2005) to allow for a symmetric Band-TAR process that allows for heteroscedastic errors and investigates monthly interest rate differentials between the U.S. and Canada, France, Germany, Japan and the UK over the period 1974-2005. Specifically, we allow for a central band within which the interest rate differential follows a unit root process and a mean reverting stationary process outside this central band. We find that TAR models can capture some of the important properties of movements in cross-country interest rate differentials over time.

2. Literature Review

Many economic time series are strongly autocorrelated and can be modeled as linear (near) unit root or $I(1)$ processes. One such series is the interest rate differential. Crowder (1995), for example, shows that cross-country interest rate differentials can be characterized by a

high degree of persistence and conditional heteroscedasticity. He finds that in most cases the null of a unit root for interest rate differentials across countries cannot be rejected.

As mentioned above, interest rate differentials are an important variable of interest to those engaged in carry trade, “a strategy where an investor borrows in a foreign country with lower interest rates than their home country and invests the funds in their domestic market, usually in fixed-income securities”¹. Ito (2002, p. 15) writes that the Japanese government has actively engaged and profited from “carrying (interest rate differential) profits from interventions during the ten year” and the “unwinding yen-carry trade positions.” Ho et al. (2005) reports that carry trade is used extensively in countries with tightly managed exchange rates, particularly in Asian economies. Chinn (2005) writes that “at the short horizon (one month, 3 months), the forex traders make plenty of money betting *against* this relationship (it's called the carry trade).”

Wadhvani (1999; p. 13) relates uncovered interest parity (UIP) to the random walk hypothesis and finds that when the interest rate differential responds less than the percentage change in exchange rates (i.e. $\beta < 1$) “carry trades make sense, because the advantage of holding the high-interest rate currency is only partially offset by a currency depreciation.”² He finds that the more evidence in favor of the random walk hypothesis, the more support for carry trade; whereas, the more evidence in support of UIP, the less potential for profit from carry trade. Chinn (2005) also finds that the greater the interest rate differential, the more likely one can profit, because this is when UIP is most likely to fail to hold. A recent Deutsche Bundesbank (2005) report examines the importance of carry trade on exchange rate dynamics and its relevance when UIP is weak. Following this logic, because of the existence of transactions costs, carry trade is unlikely to occur extensively when interest differentials are low, but is likely to be prevalent when differences between interest rates across countries increase as the potential for

profits rises. This activity leads to the interest rate differential narrowing, or mean reverting quickly and hence can be modeled using a TAR framework. For example, Naug (2003; p. 132) writes in a Bank of Norway report that “Carry traders are interested in the krone as long as the interest differential is high; changes in the differential may not matter for these traders when the differential is low.”

Studies by Caner and Hansen (2001), Enders and Granger (1998), Gonzalez and Gonzalo (1999), Gospodinov (2001, 2005), and Lanne and Saikkonen (2002), argue that the apparent (near) unit root behavior of many financial and economic time series may be the result of omitted nonlinearity. Taylor (2001) demonstrates that if the true process is a threshold, then Said and Dickey (1984) ADF unit root tests will have an autoregressive coefficient biased towards one. For example, if the autoregressive (AR) representation of the interest rate differential switches between stationary and nonstationary regimes, then ADF unit root testing procedures will have difficulty in detecting mean reversion, and false inference may occur due to the size distortions induced by the misspecification. The above studies offer a number of examples to reinforce the investigation of nonlinear specifications. Gospodinov (2005) and Ang and Bekaert (2002a,b) show that short-term interest rates can exhibit significant nonlinear behavior that depend on term structure, business cycle phenomena or the volatility ratio of long and short-term interest rates. As mentioned, we show that the nonlinear behavior of the interest rate also manifests itself in the nonlinear behavior of the interest rate differential across countries; that is, the macroeconomic phenomena responsible for regime switching in the level of interest rate do not occur simultaneous in other economies.

A central difficulty in modeling interest rates and interest differentials (e.g. at weekly or monthly frequencies) is that innovations are highly persistent and exhibit strong conditional

heteroscedasticity. One approach is to employ nonparametric methods to estimate the drift and diffusion functions and construct tests for nonlinearity. The nonparametric procedures suggested in the recent literature (Alt-Sahalia 1996; Stanton 1997), however, are not appropriate for highly persistent series and can lead to severe size distortions (Conley et. al. 1997; Pritsker 1998) and spurious results (Chapman and Pearson 2000). These estimators are biased in the extremes of the estimated function where there are only a few observations available. In addition, they depend crucially on the choice of the bandwidth (smoothing) parameter. For highly persistent data, the normal recommendations for an optimal bandwidth are not appropriate. Furthermore, Mark and Moh (2002), provide evidence for heavy tailed distributions in interest rate differentials and attribute this to “Big News.”

The estimation of regime switching (RS) models to characterize the movements of economic variables that exhibit near unit behavior has become increasingly popular in recent years. Two types of regime switching models that have been employed are Markov-switching (MS) models and Threshold Autoregressive (TAR) Models. Gonzalez and Gonzalo (1999) and Caner and Hansen (2001) have considered TAR models as alternatives to linear (near) unit root models. Their model allows for a series to have a (near) unit root in one regime while being stationary in the other. If the first order autoregressive coefficient is switching between two regimes, one that is stationary and the other non-stationary (or near unit root), then linear testing procedures will have difficulties detecting the mean reversion of the process. Caner and Hansen (2001) also derive statistical tests for testing their TAR models against linear (near) unit root models. These linear models assume a stationary threshold variable, which in practice is typically the lagged difference of the series. Lanne and Saikkonen (2002) consider TAR models that have constant stable roots smaller than unity in all regimes and in which only the intercept

term switches between different regimes. Carrasco (2002) find that tests with a threshold alternative have power against parameter instability due to structural change or Markov switching behavior.

In the international finance arena, research has employed the TAR family of models in an attempt to model the so-called purchasing power parity (PPP) puzzle in which real exchange rates exhibit short-run fluctuations and, in addition, deviations from PPP have very long half lives.³ The extant literature on the term-structure of interest rates frequently employs regime switching (RS) models (see Hamilton 1988; Sola and Driffill 1994; Bekaert, Hodrick and Marshall 2001). Regime switching models have also been applied to the level of interest rates (Gray 1996; Ang and Bekaert 2002a). One attraction of RS models is that they can accommodate some of the nonlinearities in interest rates that appear in higher order unconditional moments. For example, Gray (1996) and Bekaert, Hodrick, Marshall, (2001) show that it is only at low levels of interest rates that interest rates behave as a random walk, whereas high levels of interest rates exhibit considerable mean reversion.⁴

3. Methodology and Background

The original TAR models maintains the assumption that the data are stationary.⁵ These modeling frameworks are not able to discriminate between non-stationarity and non-linearity. Caner and Hansen (2001) develop a framework that allows for both stationary and non-stationary processes. Consider the following representation of a TAR process for a series y_t .

$$\Delta y_t = I_t \rho_1 y_{t-1} + (1 - I_t) \rho_2 y_{t-1} + \varepsilon_t, \quad (1)$$

where I_t is the Heaviside indicator function such that

$$I_t = \begin{cases} 1 & \text{if } y_{t-1} < \lambda \\ 0 & \text{if } y_{t-1} \geq \lambda \end{cases}$$

where λ = the value of the threshold and $\{\varepsilon_t\}$ is a sequence of zero-mean, constant-variance iid random variables, such that ε_t is independent of $y_j, j < t$. Petrucelli and Woolford (1984) show that sufficient conditions for the stationarity of $\{y_t\}$ are $\rho_1 < 0, \rho_2 < 0$ and $(1 + \rho_1)(1 + \rho_2) < 1$ for any value of λ . When $\rho_1 = \rho_2$, the above is referred to as a symmetric TAR model. Tong (1983, 1990) show that the least squares estimates of ρ_1 and ρ_2 have asymptotic multivariate normal distribution.

Gospodinov (2001, 2005) extends Caner and Hansen (2001) by allowing for conditional heteroscedasticity of unknown form and a GARCH(1,1) error process. He uses bootstrap methods to obtain the nonstandard distributions. The following discussion draws heavily from Gospodinov (2005). Assume a data generating process (DGP) given by the TAR model

$$y_t = I_{\{z_{t-1} < \lambda\}}(a_1 + b_1 y_{t-1}) + I_{\{z_{t-1} \geq \lambda\}}(a_2 + b_2 y_{t-1}) + \varepsilon_t \quad (2)$$

where $I_{\{q\}}$ is the indicator (Heaviside) function, z_{t-1} is a threshold variable, and λ is the threshold. Gospodinov (2005) is interested in the case when the largest AR root is close to one so he rewrites the above model as

$$\Delta y_t = \mu + \rho y_{t-1} + I_{\{z_{t-1} \geq \lambda\}}(\gamma + \phi y_{t-1}) + \varepsilon_t, t = 1, \dots, T, \quad (3)$$

where $\rho = b_1 - 1, \phi = b_2 - b_1, \mu = a_1$ and $\gamma = a_2 - a_1$. Re-parameterizing the coefficient on y_{t-1} as local-to-zero $\rho = c/T$, where $c \leq 0$ is a constant, is very helpful in analyzing the properties of the estimators and the corresponding test statistics. Let $\theta_1 = (\mu, \rho)'$ and $\theta_2 = (\gamma, \phi)'$ denote the

parameter vectors in regime 1 and 2 respectively and $\theta = (\theta_1' \theta_2')$. The null hypothesis is $H_0 : \theta_2 = 0$.

Equation (3) uses a dummy variable approach to test differences in the two regimes. If the null hypothesis of a nonlinear term (or dummy variable) can be rejected, then linearity is rejected in favor of the alternative hypothesis of a two-regime threshold model. We use a Lagrange multiplier (LM) procedure as it does not require estimation of the unrestricted model, which is sometimes difficult in nonlinear estimation.

Following Quandt (1960) and Davies (1987), a SUP-LM test can be used to evaluate the maximum value of the computed statistics. We also employ the AVE-LM statistic, following Andrews and Ploberger (1994), which is optimal against local alternatives. To introduce the estimation framework, suppose that interest rate differentials are generated from a near-integrated TAR model with errors that follow a GARCH(1,1) process

$$\begin{aligned} \Delta y_t &= \mu + \rho y_{t-1} + I_{\{z_{t-1} \geq \lambda\}} (\gamma + \phi y_{t-1}) + \sqrt{h_t} \xi_t \\ h_t &= \omega + \alpha \varepsilon_{t-1}^2 + \beta h_{t-1} \end{aligned} \quad (4)$$

with $\rho = c/T, \omega > 0, \alpha \geq 0, \beta \geq 0$. It is further assumed that the standardized errors

$\xi_t = \varepsilon_t / \sqrt{h_t}$ are *iid* with $E(\xi_t) = 0, E(\xi_t^2) = 1, E|\xi_t|^{2+\varepsilon} < \infty$ for some

$\varepsilon > 0$ and $E \ln(\alpha \xi_t^2 + \beta) < 0$. Because the limiting distributions of the test statistics are difficult to derive, Gospodinov (2001, 2005) employs Monte Carlo simulations to gain insight into the effects of heteroscedasticity on size and power. To mimic the heteroscedastic error structure of the data, he employs three different bootstrap methods; the fixed regressor bootstrap of Hansen (2000a, 2000b), a GARCH(1,1) bootstrap, and a feasible GLS (wild) bootstrap. The fixed regressor bootstrap of Hansen (2000a, 2000b) holds all the regressors including lagged

dependent variables fixed across replications and repeatedly draws from the data a large number of times. Due to its fixed structure, it can only provide a first order approximation to the finite-sample distribution of the test statistic. If we have some a priori information about the form of the heteroscedasticity, then we can model jointly the conditional mean and the conditional variance, which would presumably result in efficiency and power gains. It is a well-established empirical result that the GARCH(1,1) process provides a very good approximation to the heteroscedastic error processes of many financial variables, including interest rates. Lastly, the null hypothesis is evaluated using a wild bootstrap that allows for more general forms of heteroscedasticity than the GARCH(1,1) framework. Additional details on the bootstrap procedure can be found in appendix A. In this analysis, the data possess an unknown small sample distribution which may be better approximated by a bootstrap than an asymptotic distribution.⁶

One limitation of both the Caner and Hansen (2001) and the Gospodinov (2001, 2005) approach is that they consider only a one-sided threshold, however a two-sided threshold may exist. In this paper, we calculate a symmetric threshold. First, we extend equation (3) to allow for a two-sided symmetric threshold by replacing the threshold variable λ with $|\lambda|$. The investigation of a symmetric threshold is straightforward. We take the absolute value of the nuisance parameter and sequentially substitute it into equation (3) to find the largest SupF_T statistic.

We employ a *symmetric* Band-TAR model and let y_t from equations (2-4) equal $(i-i^*)_t$, where i is the U.S. short-term interest rate and i^* is the foreign short-term interest rate. We set the delay parameter equal to unity because for a financial variable the delay is likely to be short and

designate the threshold variable to be $|z_{t-1} = \Delta(i - i^*)_{t-1}|$. To make things more explicit, note that a symmetric TAR model of this type can be written as:

$$\begin{aligned} \Delta(i - i^*)_t &= \mu_O + \rho_2(i - i^*)_{t-1} + \varepsilon_t \text{ if } \Delta(i - i^*)_{t-1} > \lambda \\ &= \mu_I + \rho_1(i - i^*)_{t-1} + \varepsilon_t \text{ if } |\Delta(i - i^*)_{t-1}| \leq \lambda \\ &= \mu_O + \rho_2(i - i^*)_{t-1} + \varepsilon_t \text{ if } \Delta(i - i^*)_{t-1} < -\lambda \end{aligned} \quad (5)$$

The error process in (5), not shown for brevity, follows the GARCH(1,1) process as described in (4) above. The degree of mean reversion within the threshold bands is given by ρ_1 and is hypothesized to be near zero (a unit root if it is insignificantly different from zero). The degree of mean reversion in the upper and lower thresholds are assumed equal and given by ρ_2 , and is hypothesized to be mean reverting, $\rho_2 < 0$. We restrict the degree of mean reversion in the upper and lower threshold to be equal. Empirical analysis reveals that this hypothesis cannot be rejected and thus, imposing the restriction increases the power of our subsequent tests. If $\rho_1 \neq \rho_2$ the degree of mean reversion beyond the upper and lower thresholds is different from that within the threshold bands. The terms μ_I and μ_O , represent intercept terms for inside and outside the threshold bands respectively.

The threshold parameter λ , is unknown and can be endogenously determined.⁷ A threshold exists when we can reject the null hypothesis that $\rho_1 = \rho_2$. When $|\Delta(i - i^*)_t| \leq \lambda$ and $\rho_1 = 0$ then $(i - i^*)_t$ is a random walk. When $|\Delta(i - i^*)_t| > \lambda$, and $\rho_2 < 0$ then $(i - i^*)_t$ follows a stationary mean reverting autoregressive process. Note that we use the lagged one period change in the interest rate differential as the threshold variable. This specification is appropriate in the present context because the statistical inference for threshold nonlinearity is derived under the

assumption of a stationary threshold variable. Hansen (1997) presents a statistical argument for this form of adjustment. He finds that if the threshold variable is a near unit root process, then it is safer to work with the differenced threshold rather than its level. As the interest rate differential has been shown to be strongly persistent and close to a unit root process, we use the lagged change in the interest rate differential as the threshold variable.⁸

We employ the methodology of Gospodinov (2001, 2005) and allow for a GARCH(1,1) error process and test for the existence of thresholds. To allow for a symmetric threshold we employ the absolute value of the lagged change in interest rate differential as the threshold variable. This yields a symmetric band around zero. We employ both the SUP-LM and the AVE-LM tests for the existence of a threshold.

Before discussing the findings of our LM tests, it is worth noting the similarities between endogenous structural change SUP and AVE tests and threshold SUP and AVE tests. The AVE-LM and SUP-LM tests are derived from the optimality conditions of Andrews (1993) and Andrews and Ploberger (1994), and possess the same nonstandard asymptotic distributions found by Hansen (1992). Hansen (1992) shows that parameter stability tests can evaluate the existence of a cointegrating vector, and since the alternative hypothesis of a random walk in the intercept is identical to no cointegration, the test statistics are tests of the null of cointegration against the alternative of no cointegration. The average (AVE) LM and supremum (SUP) LM employed in Hansen (1992) are LM statistics for structural change in the cointegrating vector with an unknown breakpoint. These procedures have been amended to test for a threshold. Hansen (1997, 2000a) shows that the testing procedures are similar - both the endogenous structural break and the TAR procedure use nuisance parameters and the SUP or AVE LM statistics as test criterion for evaluating/determining the parameter that minimizes the error in the estimated

equations. The AVE-LM statistic tests whether the specified model is a good model that captures a stable relationship. A large test statistic implies that there is a break or threshold. The SUP-LM is appropriate for testing a swift shift in regime and its power is also concentrated on the distant alternative hypothesis. We use this SUP-LM statistic to evaluate differences in the error between models. A large test statistic implies that we can reject a linear specification.

4. Empirical Results

The data employed in this paper are monthly short-term, money market interest rates for Canada, France, Germany, Japan, the UK and U.S. obtained from the IFS database. We use the U.S. as the benchmark and study interest differentials vis-à-vis the U.S. The sample period for the U.S., Germany, Japan, and the UK is 1974.01-2005:11; for France the sample period is 1974.1-1998.12 and for Canada the sample period is 1975.1-2005.11. We begin the data in 1974 to approximately coincide with the fall of the Bretton Woods system. Using data that begin substantially earlier would likely have been affected by capital controls, which were endemic in the 1950s and 1960s. To minimize the effect of large outliers on our data, we follow Balke and Wohar (1998) and regress the interest rate differential for each economy on its first lag, and remove any observations associated with residuals that are more than four standard deviations from zero. This resulted in only a small number of excluded observations.

Table 1 reports Elliot, Rothenberg and Stock (1996) ADF-GLS test statistics to determine whether the level of the interest rate and interest rate differential across each country follow a unit root process. The selection of lagged difference terms in the ADF regression is chosen using the AIC criteria. The results indicate that a unit root process cannot be rejected at the 5% level of

significance for either the level of interest rates or the interest rate differential, except for the UK-U.S. differential. The results (not reported) when Germany is the benchmark also indicate that the null of unit root cannot be rejected. Enders and Granger (1998) show that tests for unit roots and cointegration all have low power in the presence of asymmetric adjustment (such as a TAR process). We next illustrate the existence of heteroscedasticity in the interest rate differentials and then investigate the existence of nonlinear threshold behavior.

Table 2 presents some preliminary diagnostic tests examining whether the residuals from an AR(1) process of the interest rate differentials follow a homoskedastic or heteroskedastic process. The reported p-values from an ARCH-LM test of the residuals (row 1) indicate that the null of a homoskedastic error process is strongly rejected. There is also strong evidence of kurtosis (row 3) in the residuals (as the statistic exceeds 3) in all cases, as well as some negative skewness (row 2) for three of the five cross-country interest rate differentials. The interest rate differentials exhibit leptokurtic behavior in all cases. In modeling the residuals as an ARCH and GARCH process, we note that the ARCH (row 4) and GARCH (row 5) parameters are very significant, indicating the volatility of interest rate differentials is persistent and can be modeled as a GARCH (1,1). There is also evidence that the GARCH process is integrated or near integrated as the parameters sum to unity. A TAR process may be a potential explanation for the high GARCH persistence, since neglected nonlinearity can induce near unit root volatility.

We begin our investigation of threshold behavior in interest rate differentials by examining whether the interest rate differential can be modeled as a symmetric threshold model with the interest rate differential exhibiting (near) unit root behavior inside the band and exhibiting mean reverting behavior outside the band. Rows 1-3 in Table 3 report the homoskedastic fixed regressor (HOM-LM), a feasible GLS (HET-LM) and the GARCH(1,1)

(GARCH-LM) bootstrap test statistics, respectively, following Gospodinov (2001, 2005). We are principally concerned with the results based on a GARCH(1,1) error process, and thus, our attention will be on the GARCH-LM test. While the HOM-LM (row 1) and HET-LM (row 2) tests give mixed results, the GARCH-(SUP) LM (row 3) test rejects the null of no threshold in favor of a symmetric threshold for all five interest rate differentials (although the U.S.-France interest rate differential is rejected only at the 10% level). The results of the LM tests indicate that allowing for a heteroscedastic error process can be important in detecting a threshold. Assuming a homoskedastic process may lead one to falsely infer that no threshold exists. We also report the percentage reduction in the sum of squared residuals (SSR) from an AR to a TAR model; results show that the decrease in the SSR ranges between 3-8%, which is significant at the 5% level in all five cross-country interest rate differentials.

Row 6 and 7 of Table 3 report the value of ρ_1 (corresponding to the measure of persistence inside the symmetric threshold bands) and ρ_2 (corresponding to the measure of persistence outside the threshold bands); for conciseness, we do not report the intercept terms. We also report the standard errors and designate with an asterisk (*) our coefficients when they are significantly different from unity using bootstrapped t-statistics. For all five economies, results cannot reject $\rho_1=1$, but can reject at the 5% level that $\rho_2=1$. Inspection of the coefficients reveals substantial differences between ρ_1 and ρ_2 for Canada, France and UK, and reflects wide differences in mean reversion and half-life adjustment. For Germany and Japan, the coefficient differences are smaller, but nonetheless economically significant and meaningful; e.g., for both these economies, there is essentially little or no mean reversion inside the band, while outside the band the approximate half life adjustment is 19 months for Germany and 33 months for Japan. These results support the two regime TAR specification for all five countries –a stationary mean

reverting process for interest rate differentials outside the bands and a unit root process inside the bands.

The total percentage of observations outside the threshold bands is reported in row 4 of Table 3. For four of the five economies, the interest rate differential follows a mean reverting process only about 15% of the time, while for Japan 30% of the monthly observations mean revert. The bottom row reports the threshold bands. For all five economies; their values lie between 2.1-3.7%. Observations outside the bands imply large differentials and mean reverting interest rate differentials.

We next turn to examining how well two nonlinear models of interest rate differentials perform in terms of out-of-sample forecasting relative to an AR(1) model specification. We examine our Band-TAR model as well as the commonly used exponential smooth TAR (ESTAR) model. We use the mean square forecast error (MSFE) criteria for two out-of-sample periods – a five year (60 observations) period and a ten year period. In the first and second columns of Table 4 for every country, we report the ratio of the MSFE associated with the 1-month to 12-month step ahead forecasts, for an ESTAR relative to an AR(1) specification and our symmetric Band-TAR model relative to an AR(1) specifications, respectively.⁹ These forecasts are computed by recursively updating the sample. Statistics less than one for these columns imply that the ESTAR and the symmetric Band-TAR models produce lower forecast errors than the AR(1) benchmark. Table 4 presents the results, and we bold the statistics that produce the lowest MSFE for each country's monthly forecast.

For the 5-year out-of-sample forecasting period, we find that our symmetric Band-TAR model outperforms the AR(1) benchmark in Canada, France and Germany for all forecast periods. Note that the ratios are considerably below unity for these three countries for all forecast

months and imply on average (across the twelve months) an 11%-13 reduction in forecast errors. Due to the assumption of heteroscedasticity, standard errors and significance are not presented; however, using a homoskedastic fixed regressor bootstrap, we can reject the null of equal MSFE for nearly all months. Given the large reductions in MSFE, it is reasonable to assume that most heteroscedastic specifications will likely indicate significance and therefore increased forecasting ability of a symmetric Band-TAR specification compared to an AR(1) model. The Band-TAR model also produces modest reductions for several months for Japan and U.K. Results additionally demonstrate that the ESTAR specifications perform very well with ratios well below one for Canada, France and Germany. In contrast, for Japan and UK, the ESTAR model leads to higher MSFE, as the results in Table 4 shows that the MSFE ratios are consistently above one.

Results for the 10-year out-of-sample period show that our Band-TAR model produces ratios substantially below one for Germany, and often in Canada, France, Japan and UK. A homoskedastic bootstrap (which as mention can produce an approximation for inference testing) also rejects the null of equal MSFE for Germany and for several months for Canada, France and Japan. The ESTAR specification also performs well, and consistently outperforms the AR(1) benchmark in Canada and Germany, and often in Japan. Note, it is difficult to discern between either the Band-TAR or ESTAR models in out-of-sample forecasts as these models are close to each other in terms of their performance. We interpret these results in favor of a nonlinear specification and believe that our evidence thus far lends credence to the Band-TAR model. More specifically, the in-sample evidence presents substantially different speeds of adjustment depending on whether the differential is inside the band or outside. Financially market adjustment is likely to be fast, not slow or smooth, and carry trade is likely to be a function of

transactions costs that might be better approximated by a Band-TAR model with a band of inactivity within which interest rates follow a random walk process, while following a stationary mean reverting process outside the bands.

Figures 1, panels A, B and C, plots the interest rate differential for Canada, Germany and UK and their respective bands estimated from our symmetric Band-TAR model. With respect to the U.S.-Germany linkage, it is clear from the short-lived large (but narrow) spikes in the first half of the sample, that when the change in the interest rate differential in the previous period is large there is strong mean reversion. The large negative duration was more long-lasting (about 10 months), and hence carry trade was profitable for some time, but ultimately this profitability was transitory. The U.S.-UK panel also illustrates short lived large and narrow spikes in the first half, that were quickly mean reverting, again implying that carry trade opportunities are not very persistent. The U.S.-UK panel also has a more long-lasting period (approximately the same time as the German differential), which was also eventually diminished.

5. Conclusions

The spread between domestic and foreign interest rates is a relevant variable to central bankers as well as investors in the foreign exchange market who are engaged in currency carry trade. Studies find that the time series behavior of interest rate differentials across countries is characterized by high persistence and heteroscedasticity when the data frequency is monthly or higher. One explanation for the apparent (near) unit root behavior is that the interest rate differential follows a nonlinear TAR process, where small deviations are persistent, but too small for carry trade to be profitably conducted due to uncertainty and/or transactions costs. Larger deviations, on the other hand, that exceed a threshold allow for and encourage profitable carry trade, which then induce mean reverting interest rate differentials. Hence, a band or threshold

may exist around interest differentials, and the degree of mean-reversion depends on the size of the previous change in the interest rate differential.

In this paper we examine the time series properties of monthly cross-country interest rate differentials between the U.S. and Canada, Germany, France, Japan and the UK, over the period 1974-2005, employing a nonlinear symmetric band threshold autoregressive model that allows for heteroscedasticity in the error process. We find the existence of a central band of “inaction” where the interest rate differential follows a unit root process for small interest rate differentials within the threshold bands, while exhibiting mean reversion behavior when outside the symmetric bands. Using one-month interest rate differential data, we significantly reject linearity in favor of the alternative hypothesis of a symmetric Band-TAR model that allows for heteroscedastic errors.

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Available at www.bankofengland.co.uk/publications/speeches/1999/speech53.pdf.

Endnotes

¹ Quote from www.Investopedia.com/terms/c/currencycarrytrade.asp .

² Wadhvani is referring to the regression of the change in the spot exchange rate on the differential between foreign and domestic interest rates (i.e. $\Delta s_{t+k} = \alpha + \beta(i_t^* - i_t) + v_{t+k}$).

³ In modeling real exchange rates, various RS models have been employed; Obstfeld and Taylor (1997) and A. Taylor (2001) employ a Band-TAR model. Michael, Nobay, and Peel (1997) and Taylor, Peel, and Sarno (2001) consider an exponential smooth transition autoregressive (ESTAR) model. Both of these models are characterized by symmetric adjustment. Bergman and Hansson (2005) estimate a two-state Markov-switching AR model of real exchange rates.

⁴ Cecchetti, Lam and Mark (1993) and Garcia (1998) show that single-regime models are econometrically rejected in favor of their regime-switching counterpart. There are a number of studies that document non-linear behavior in interest rates, including, Pflann, Schotman and Tscherning (1996), Ait-Sahalia (1996), Conley, Hansen and Luttmer, and Scheinkman (1997), Boudoukh, Richardson, Stanton, and Whitelaw (1997), Stanton (1997), Ahn and Gao (2000), Gospondinov (2001), and Ang and Bekaert (2002a, 2002b), among others. Balke and Wohar (1998), employ a stationary three-regime TAR model, and find that the time series properties of deviations from covered interest parity follow a TAR process, where arbitrage costs imply faster reversion the larger the deviations from covered interest parity are outside the threshold bands. Furthermore, Saikkonen and Ripatti (1999) recently suggests that the TAR model is the appropriate framework for testing and modeling expectational economic relationships in the presence of "peso effects".

⁵ Tong (1990) provides a survey of the statistical and dynamic properties of the TAR process.

Hansen (1997, 1999) provides a review of some of the recent advances in making inferences in TAR models.

⁶ A useful survey of the relevance and importance of heteroscedasticity is McAleer and Oxley, (2002), while Ruiz and Pasual (2002) surveys the application of bootstrap methods to financial time series. Davidson and MacKinnon (1999) provides a theoretical framework for the bootstrap. Li and Xiao (2001) shows that the bootstrap provides reasonably good approximation to the distribution of integrated series.

⁷ The threshold takes on values in the interval $\lambda \in \Lambda = [\lambda, -\lambda]$ where λ and $-\lambda$ are picked so that $P(Z_{t-1} \leq -\lambda) = \pi_1 > 0$ and $P(Z_{t-1} \leq \lambda) = \pi_2 < 1$. Generally, π_1 and π_2 are treated as symmetrical so that $\pi_2 = 1 - \pi_1$. This imposes the restriction that no regime has less than $\pi_1\%$ of the total sample. The choice of π_1 in practice is guided by the consideration that each regime must have a sufficient number of observations to adequately identify the regression parameters. We follow Andrews (1993) and select $\pi_1 = 0.15$ and $\pi_2 = 0.85$. We also conduct analysis for $\pi_1 = 0.05$ and $\pi_2 = 0.95$, and $\pi_1 = 0.10$ and $\pi_2 = 0.90$. The results are not qualitatively different.

⁸ Enders and Granger (1998) and Enders and Siklos (2001) show that this specification is especially relevant when the adjustment is such that the series exhibit more momentum in one direction than the other.

⁹ We thank an anonymous referee for suggesting this forecasting exercise.

Table 1: ERS ADF Unit Root Tests

	CA	FR	GE	JP	UK	U.S.
i^* (Level)	-1.33	-1.73	-2.83	-2.50	-2.68	-1.70
α	-0.02	-0.02	-0.03	-0.01	-0.05	-0.01
Lag length	5	3	12	6	9	11
$i^{us} - i^*$	-2.34	-2.36	-1.73	-2.84	-3.23*	
α	-0.08	-0.05	-0.02	-0.04	-0.09	
Lag length	7	2	11	7	11	

ERS is Elliot, Rothenberg and Stock ADF-GLS test statistics.

* Reject at 5%. The coefficient α is the sum of the autoregressive coefficients.

Sample period for Germany, Japan, United Kingdom and United States: 1974.1-2005.11.

Sample period for Canada: 1975.1-2005.11 and France: 1975:01-1998:08

Table 2: Preliminary Diagnostic Tests

Interest Rate Differential	$i^{\text{US}} - i^{\text{CA}}$	$i^{\text{US}} - i^{\text{FR}}$	$i^{\text{US}} - i^{\text{GE}}$	$i^{\text{US}} - i^{\text{JP}}$	$i^{\text{US}} - i^{\text{UK}}$
1. ARCH-LM (p-value)	[0.000]	[.006]	[0.000]	[0.000]	[0.000]
2. Skewness	-0.406	-.234	-0.062	-0.062	-0.406
3. Kurtosis	10.506	23.882	22.314	22.314	10.506
4. ARCH parameter	0.207**	0.318**	0.332**	0.332**	0.207**
(S.E.)	(0.054)	(0.056)	(0.030)	(0.030)	(0.054)
5. GARCH parameter	0.787**	0.689**	0.738**	0.738**	0.787**
(S.E.)	(0.045)	(0.052)	(0.030)	(0.030)	(0.045)
Variance: Hi 3 Year MA	7.792	3.624	8.304	5.272	12.685
Low 3 Year MA	.029	0.049	0.021	0.014	0.062

Table 3 Interest Differential Adjusting for Heteroskedasticity: Symmetric Thresholds

$$\Delta y_t = \mu + \rho y_{t-1} + I_{\{z_{t-1} \geq \lambda\}}(\gamma + \phi y_{t-1}) + \varepsilon_t, t = 1, \dots, T,$$

$$h_t = \omega + \alpha \varepsilon_{t-1}^2 + \beta h_{t-1}$$

where $y = (i - i^*)$ and $|z| = \Delta(i - i^*)$

Interest Rate Differentials	Y=i^{US}-i^{CA}	y=i^{US}-i^{FR}	y=i^{US}-i^{GE}	y=i^{US}-i^{JP}	Y=i^{US}-i^{UK}
LM tests for Symmetric					
Threshold					
1. HOM Sup	2.11	14.33	17.38	2.53	23.85*
Ave	0.60	7.92	6.08	0.62	18.06*
2. HET Sup	6.23	5.35	5.17	2.46	21.62**
Ave	5.07*	6.329	2.59	0.93	14.63**
3. GAR Sup	9.52*	6.87 ⁺	13.84*	14.54**	31.45**
Ave	3.67 ⁺	3.33 ⁺	5.96*	4.43**	16.60**
4. % of obs. in outer regimes	15	15	17	30	15
5. % reduction SSR	3.7**	4.9**	5.0**	3.2*	8.1
6. ρ_1 (inside the bands)	0.977	0.995	0.998	1.001	0.950
(S.E.)	(0.013)	(0.019)	(0.005)	(0.007)	(0.027)
7. ρ_2 (out side the bands)	0.748*	0.854**	0.967**	0.979**	0.806**
(S.E.)	(0.110)	(0.041)	(0.009)	(0.007)	(0.067)
8. Threshold Value λ	2.73	3.61	3.66	2.93	2.11

** Significant at 1%, *significant at 5%, +Significant at 10%. LM p-values. H_0 is no conditional heteroscedasticity. ARCH p values of GARCH(1,1). HOM is the fixed regressor bootstrap under homoscedasticity, HET uses a bootstrap that allows for a general form of heteroscedasticity (see Hansen 2000b). HET employs a bootstrap under a GARCH(1,1) specification of the residuals (see Gospodinov 2001). SupLM and AveLM statistics reported. Inference conducted in rows 6 and 7 are for the null hypothesis that the coefficient is equal to unity.

Table 4: Ratio of MSFE for 5 and 10 Year Out-of-Sample Forecasts

5 Year Step-Ahead	CA		FR		GE		JP		UK	
	$\frac{ESTAR}{AR}$	$\frac{TAR}{AR}$	$\frac{ESTAR}{AR}$	$\frac{TAR}{AR}$	$\frac{ESTAR}{AR}$	$\frac{TAR}{AR}$	$\frac{ESTAR}{AR}$	$\frac{TAR}{AR}$	$\frac{ESTAR}{AR}$	$\frac{TAR}{AR}$
1	0.684	0.878	0.893	0.892	0.904	0.878	1.032	0.966	1.029	1.032
2	0.637	0.896	0.848	0.911	0.874	0.832	1.068	0.985	1.025	0.999
3	0.635	0.849	0.791	0.881	0.850	0.845	1.066	1.018	1.053	0.949
4	0.647	0.838	0.825	0.877	0.861	0.853	1.096	0.993	1.012	0.948
5	0.674	0.874	0.812	0.870	0.843	0.849	1.084	1.034	1.025	0.956
6	0.695	0.894	0.779	0.871	0.829	0.845	1.129	1.024	1.081	0.950
7	0.719	0.883	0.798	0.853	0.849	0.865	1.129	0.974	1.034	0.923
8	0.741	0.892	0.791	0.902	0.844	0.872	1.145	0.994	1.036	0.964
9	0.761	0.903	0.768	0.857	0.847	0.899	1.156	1.040	1.062	0.955
10	0.782	0.908	0.785	0.851	0.840	0.909	1.163	1.018	1.095	0.994
11	0.794	0.926	0.775	0.827	0.846	0.917	1.159	1.013	1.049	1.002
12	0.807	0.931	0.775	0.809	0.852	0.931	1.192	1.003	1.065	0.998

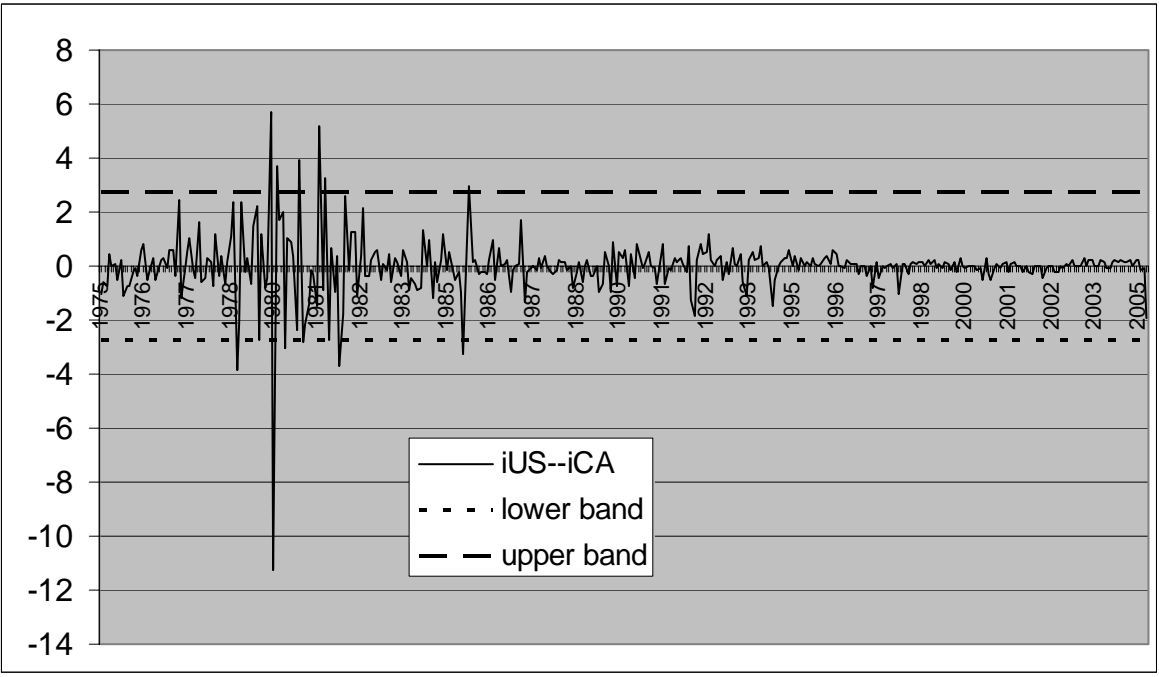
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10 Year Step-Ahead	CA		FR		GE		JP		UK	
	<u>ESTAR</u> AR	<u>TAR</u> AR	<u>ESTAR</u> AR	<u>TAR</u> AR	<u>ESTAR</u> AR	<u>TAR</u> AR	<u>ESTAR</u> AR	<u>TAR</u> AR	<u>ESTAR</u> AR	<u>TAR</u> AR
1	0.684	1.191	1.240	1.020	0.903	0.868	0.930	0.986	1.010	1.003
2	0.639	1.020	1.241	1.040	0.881	0.881	0.948	1.108	1.034	0.995
3	0.631	0.967	1.243	1.046	0.879	0.844	0.943	1.065	1.062	0.987
4	0.648	0.895	1.251	1.054	0.874	0.805	0.961	1.058	1.100	0.939
5	0.670	0.880	1.211	1.036	0.881	0.783	0.957	1.019	1.131	0.976
6	0.696	0.888	1.190	1.025	0.872	0.768	0.977	1.021	1.169	1.002
7	0.716	0.875	1.156	0.995	0.879	0.773	0.984	0.988	1.200	0.983
8	0.740	0.899	1.133	0.976	0.875	0.794	0.995	0.966	1.195	1.009
9	0.762	0.904	1.127	0.969	0.874	0.796	1.003	0.973	1.223	1.005
10	0.781	0.919	1.106	0.951	0.878	0.824	1.024	0.918	1.270	1.009
11	0.794	0.937	1.109	0.958	0.871	0.822	1.029	0.963	1.324	1.013
12	0.810	0.946	1.092	0.943	0.878	0.837	1.041	0.934	1.309	1.022

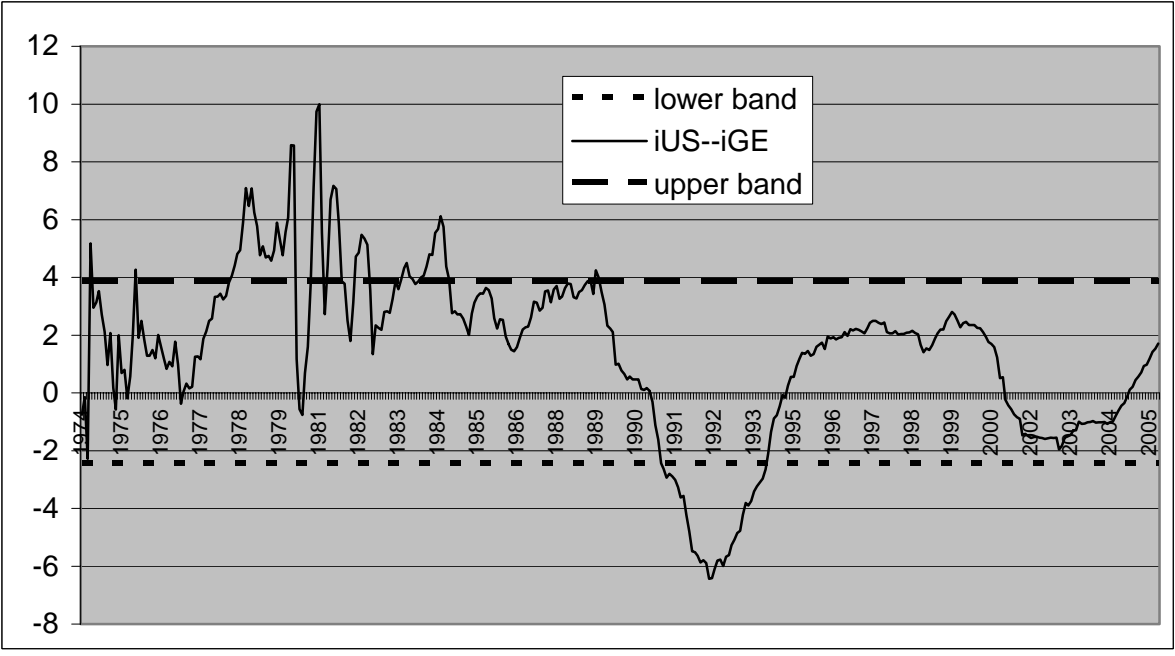
Statistics in bold are the lowest MSFE for that month.

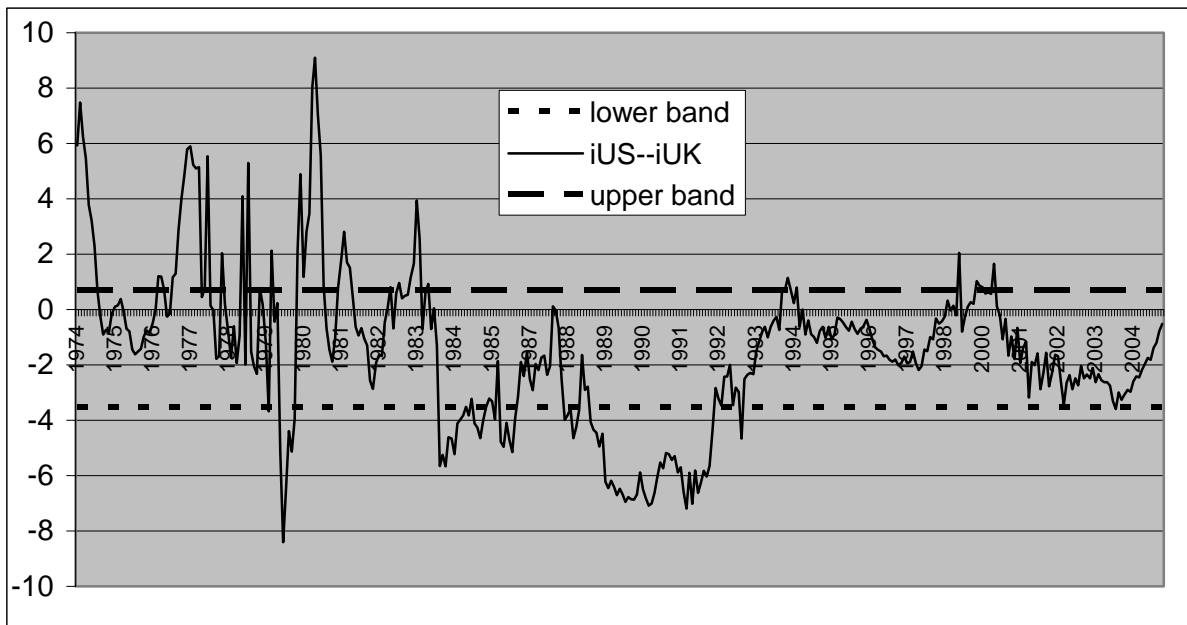
Figure 1

Panel A: U.S.-Canada Interest Differentials and Bands



Panel B: U.S.-Germany Interest Differentials and Bands



Panel C: U.S.-United Kingdom Interest Differentials and Bands

Appendix A TAR with GARCH Errors

This appendix draws from Gospodinov (2001 pp. 6-7). Since many authors have shown that the GARCH(1,1) is a good approximation to the heteroscedastic error process of many financial variables, one can increase efficiency and power by jointly modeling the conditional mean and variance as a GARCH(1,1). To obtain the finite sample critical values, we first estimate using quasi ML the threshold assuming a GARCH(1,1) To introduce the main results, suppose that interest rate data were generated from the near-integrated TAR with errors that satisfy Assumption 1 and follow a GARCH(1,1) process

$$\begin{aligned} \Delta y_t &= \mu + \rho y_{t-1} + I_{\{z_{t-1} \geq \lambda\}} (\gamma + \phi y_{t-1}) + \sqrt{h_t} \xi_t \\ h_t &= \omega + \alpha \varepsilon_{t-1}^2 + \beta h_{t-1} \end{aligned} \tag{A.1}$$

with $\rho = c/T$, $\omega > 0$, $\alpha \geq 0$, $\beta \geq 0$. It is further assumed that the standardized errors

$\xi_t = \varepsilon_t / \sqrt{h_t}$ are iid with $E(\xi_t) = 0$, $E(\xi_t^{2+\varepsilon}) < \infty$ for some $\varepsilon > 0$ and $E \ln(\alpha \xi_t^2 + \beta) < 0$.

The asymptotic representations of the standardized quantities

$T^{-1/2} \sum_{t=1}^T A_t^1$, $T^{-1} \sum_{t=1}^T A_t^2$, $T^{-1/2} \sum_{t=1}^T B_t^1$ and $T^{-1} \sum_{t=1}^T B_t^2$, are difficult to derive due to the nonlinear estimation of GARCH models. As for the heteroscedasticity-robust LM test, we only conjecture that under the null of no threshold effect in model (A.1), the distribution of the $SupF_T$ test for linearity can be reasonably approximated by a limiting distribution obtained under the assumption of conditionally homoskedastic errors.

Alternatively, conditional on the data, we can use bootstrap methods to approximate the finite sample critical or p -values of the test statistic. Assume that there exists a limiting distribution $sup F$ such that $sup F_T$ converges weakly to $sup F$ as $T \rightarrow \infty$. Then, the bootstrap p -value of the $SupF_T$ test is approximated through the following procedure. First, estimate the

GARCH(1,1) by quasi ML and compute the test statistic $SupF_T$. Calculate the standardized

residuals under the null $\hat{\xi}_t = \hat{\varepsilon}_t / \sqrt{\hat{h}_t}$. Since ξ_t are assumed to be *iid*, then we can resample

with replacement directly from their empirical distribution function to obtain the sequence $\{\xi_t^*\}$.

Then, for some initial conditions h_0^*, ξ_0^* and y_0^* , the bootstrap series $\{y_t^*\}$ is constructed

recursively under the null from

$$\begin{aligned} h_t^* &= \hat{\omega} + (\hat{\beta} + \hat{\alpha}\xi_{t-1}^{*2})h_{t-1}^* \\ y_t^* &= \hat{\mu} + (1 + \hat{\rho})y_{t-1}^* + \sqrt{h_t^*}\xi_t^* \end{aligned}$$

This algorithm is repeated B times and each time the statistic $SupF_T^*$ is computed. Then, the p-

value of the test is given by the probability $\Pr\{SupF_T^* \geq SupF_T \mid \theta = \hat{\theta}\}$. Lastly, we evaluate the

null hypothesis using a feasible GSL bootstrap that allows for more general forms of

heteroscedasticity than the GARCH(1,1).